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Faddeev-Yakubovsky and Jacobi-no-core-shell model results for light hypernuclei

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- Motivation
- Λ-Σ conversion
- A=3,4 hypernuclei
- Jacobi-NCSM and SRG-evolved interactions
- A=3-7 hypernuclei using SRG-evolved interactions
- Conclusions & Outlook

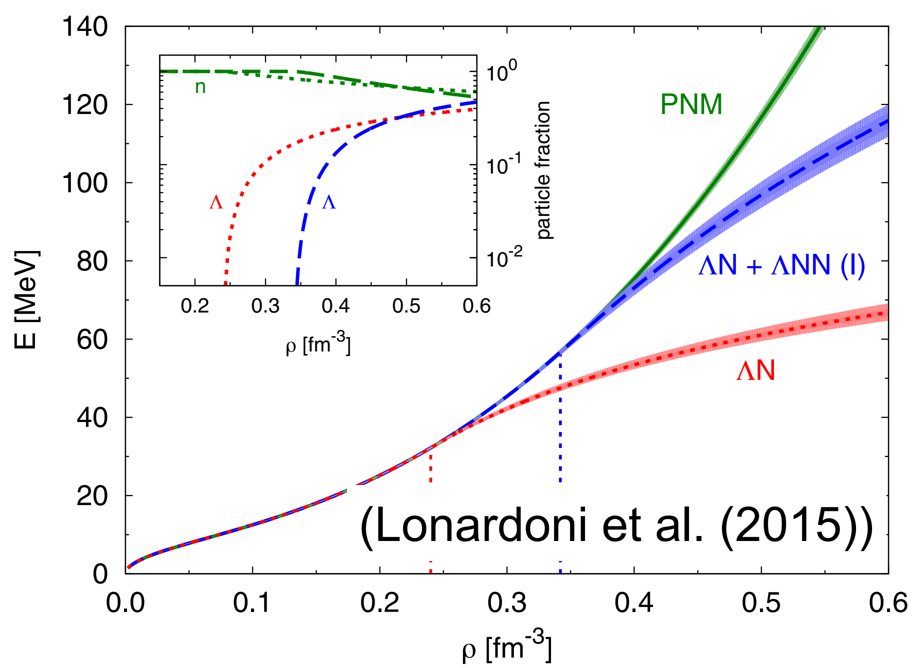
in collaboration with Johann Haidenbauer, **Hoai Le**, Susanna Liebig, Ulf Meißner

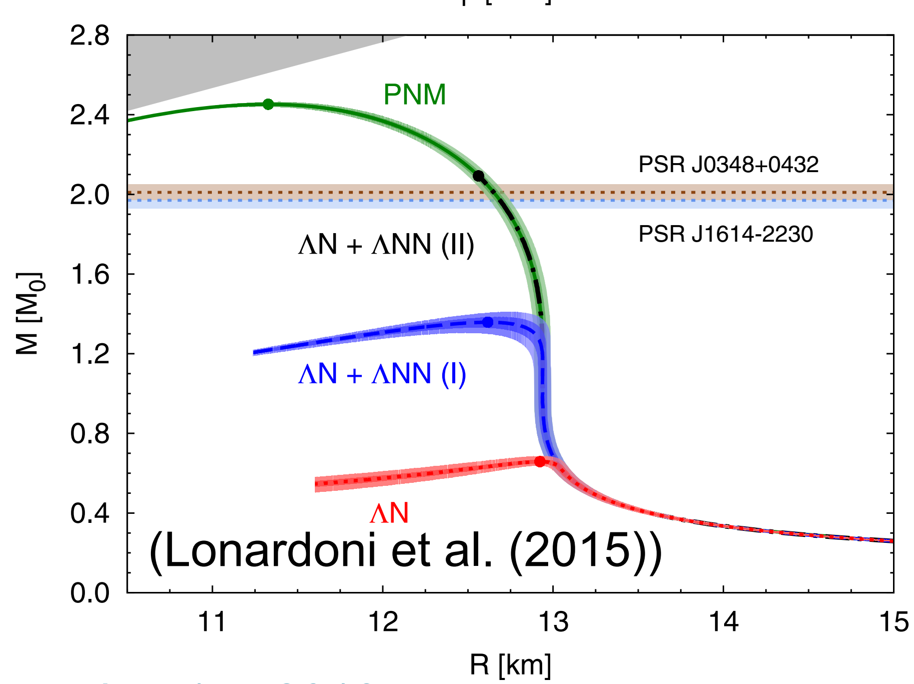
November 1st, 2018

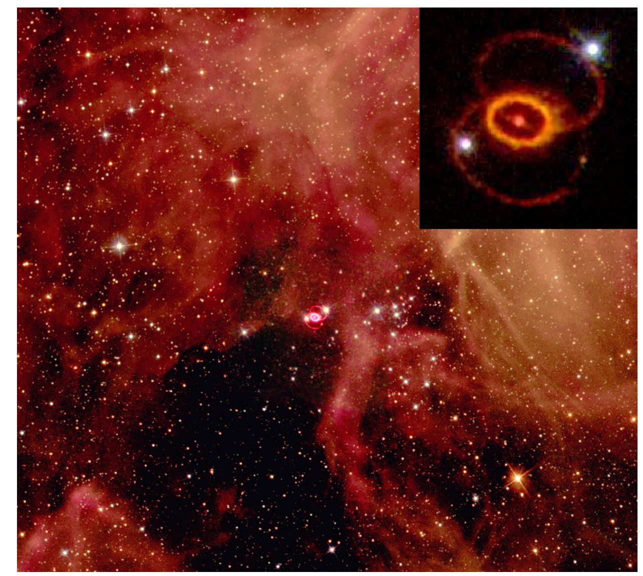
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

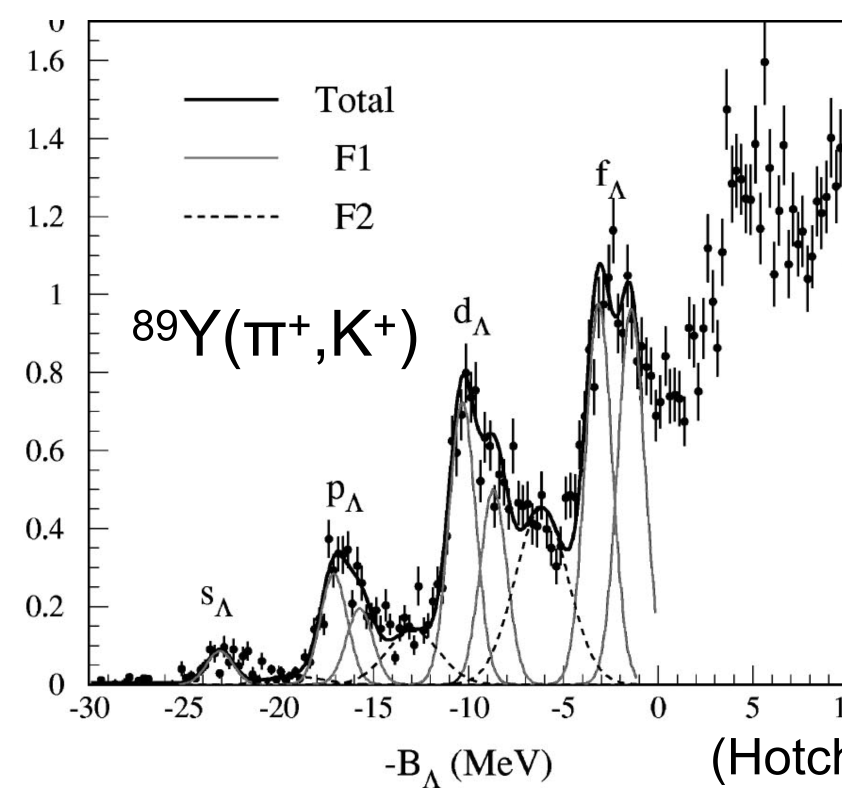
- „phenomenologically“
  - *hyperon contribution to the EOS, neutron stars, supernovae*
  - *Λ as probe to nuclear structure*







(SN1987a, Wikipedia)

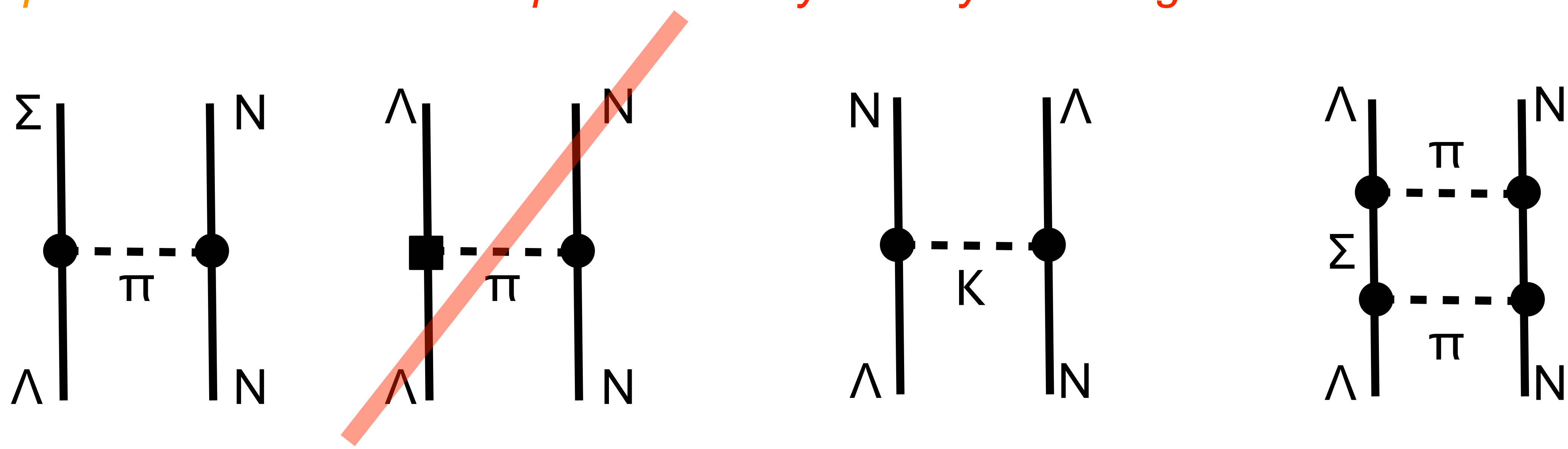


(Hotchi et al. (2001))

Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

- conceptually
  - ***Λ-Σ*** *conversion process is long-range part of the interaction (assuming isospin conservation)*
  - *experimental access to explicit chiral symmetry breaking*



suppressed by
isospin symmetry (CSB!)

$m_K \approx 500$ MeV

But it is difficult to pin down the properties of YN interactions, …

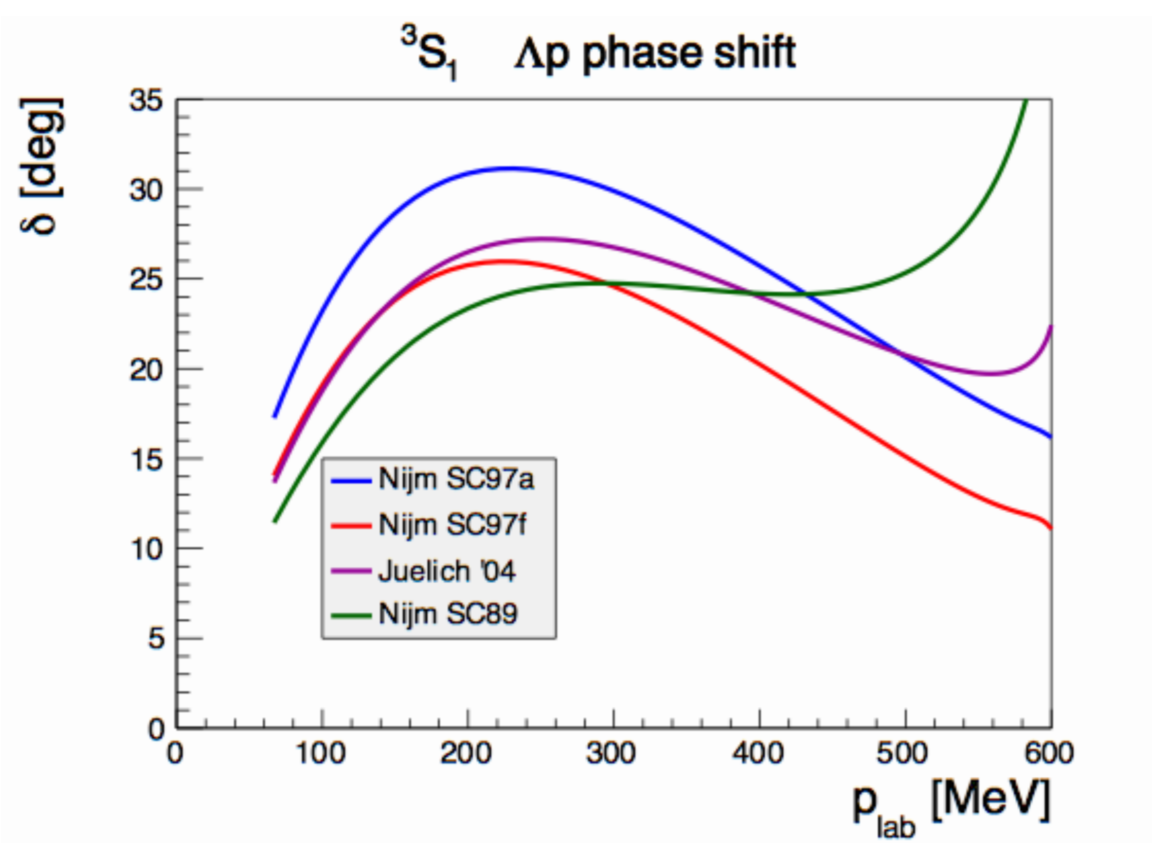
Hypernuclear interactions

37 YN data, no YN bound state, large uncertainties → no partial wave analysis possible

Recipe for more than 40 years:

- *extend a OBE exchange model for the NN interaction*
- *assume flavor SU(3) symmetry*
- *break flavor SU(3) symmetry where it seems appropriate*

→ several YN interaction models (Jülich 89/04, Nijmegen 89/97a-f, ESC, …)
describe all YN data **more than perfectly**, but are not phase equivalent

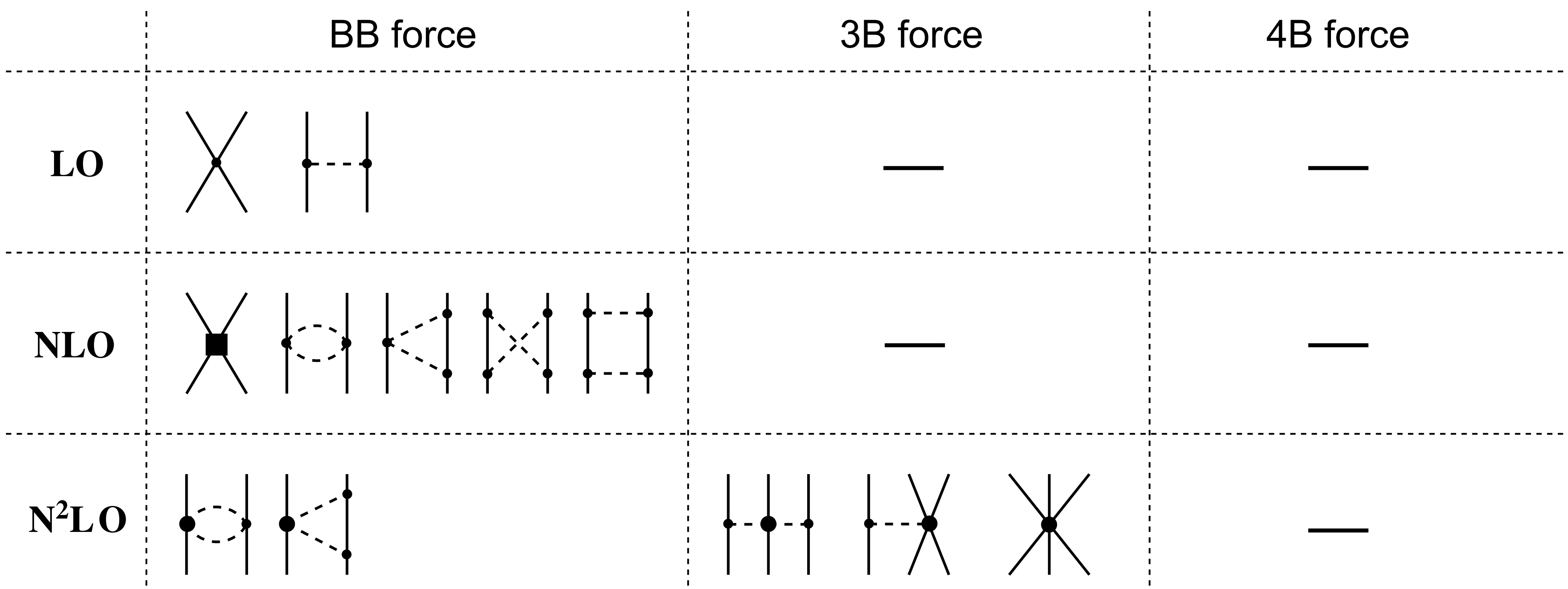


	$^1a(\Lambda p)$ [fm]	$^3a(\Lambda p)$ [fm]
SC97a	-0.7	-2.15
SC97b	-0.9	-2.11
SC97c	-1.2	-2.06
SC97d	-1.7	-1.93
SC97e	**-2.1**	**-1.83**
SC97f	**-2.5**	**-1.73**
SC89	**-2.6**	**-1.38**
Jülich ´04	**-2.6**	**-1.73**

Chiral NN & YN interactions

additional constraints required (only 37 data, but 23 parameters at NLO)

data too sparse to uniquely determine the short range LECs!



5 YN short range parameters

23 YN short range parameters

(adapted from Epelbaum, 2008)

→ we have **two** realization for the YN interaction at NLO with different assumptions on the LECs

(J. Haidenbauer et al., 2013 & work in progress)

Chiral YN interactions

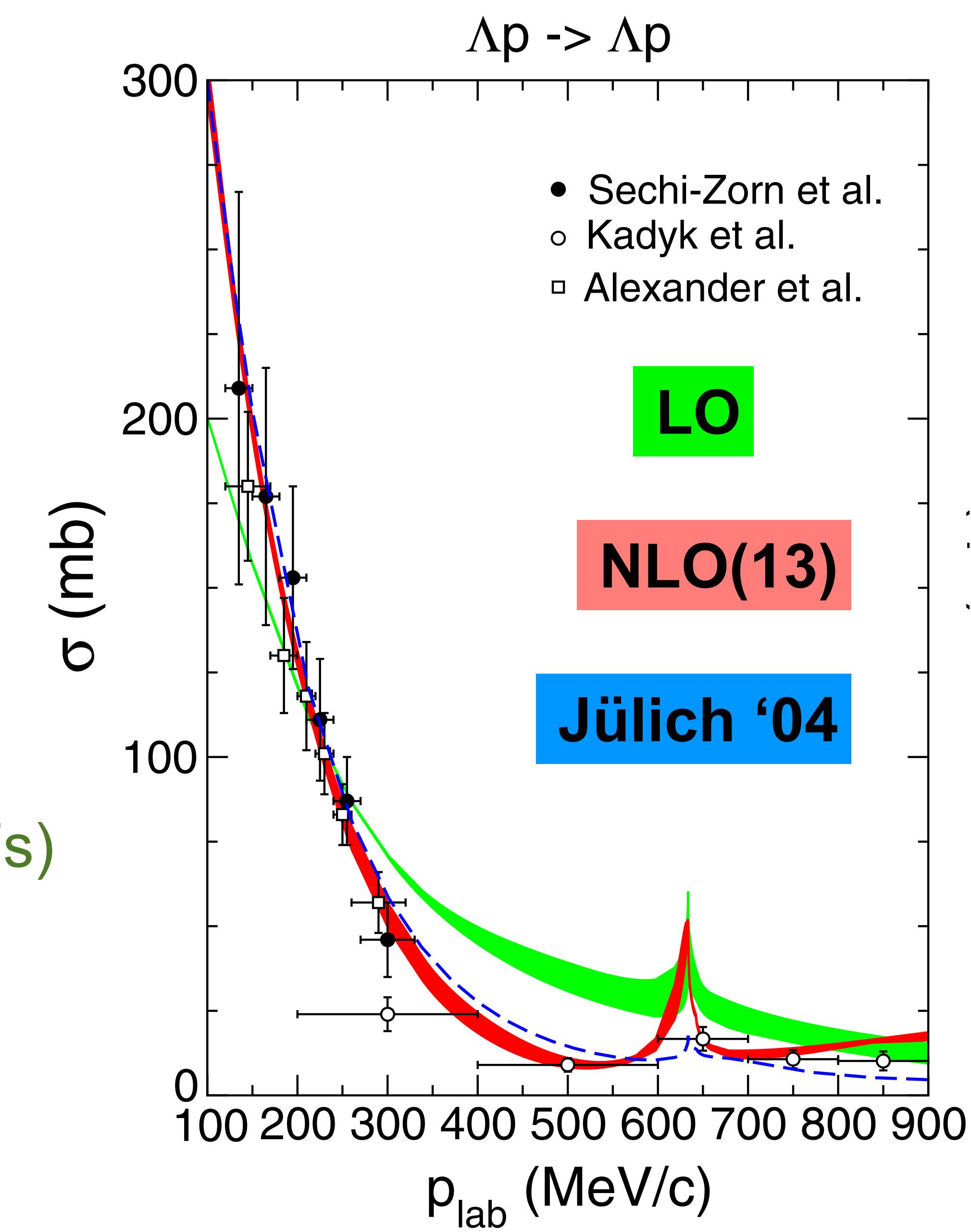
Additional constraints are required that are

not required by power counting

should be relaxed in future to explore YN realm

- SU(3) broken by physical m_π, m_K, m_η
- but: no SU(3) breaking in F_π, F_K, F_η"
- but: "minimize" P-waves and $^1P_1$-$^3P_1$ mixing
  to determine P-wave counter terms

- cutoff dependence can be studied
  to get first estimate of higher orders (including 3BFs)
  $\Lambda \approx 450 \ldots 700$ MeV



(J. Haidenbauer et al., 2013)

- different versions of the YN interactions:
  SU(3) symmetry used for contact interactions but
  1. ***LO:*** *NN constraints are not used*
  2. ***NLO(13):*** *NN constraints are not used (best χ^2)*
  3. ***NLO(15):*** *NN constraints are used for subleading contact interaction (χ^2 increases slightly)*

Hypernuclei - calculations

To further constrain the interactions, we need techniques that reliable predict hypernuclei binding energies based on various interactions.

For complex hypernuclei/hypernuclear matter

- shell model: Millener, Hungerford, Gal (2016), …
- cluster models: Hiyama (2012), …
- density functional theory: Lu, Zhao, Zhou (2011), …
- AFDMC w/o Λ-Σ conversion: Lonardoni et. al. (2015), ..

These approaches allow one to study very complex systems and **connect the results for different hypernuclei**

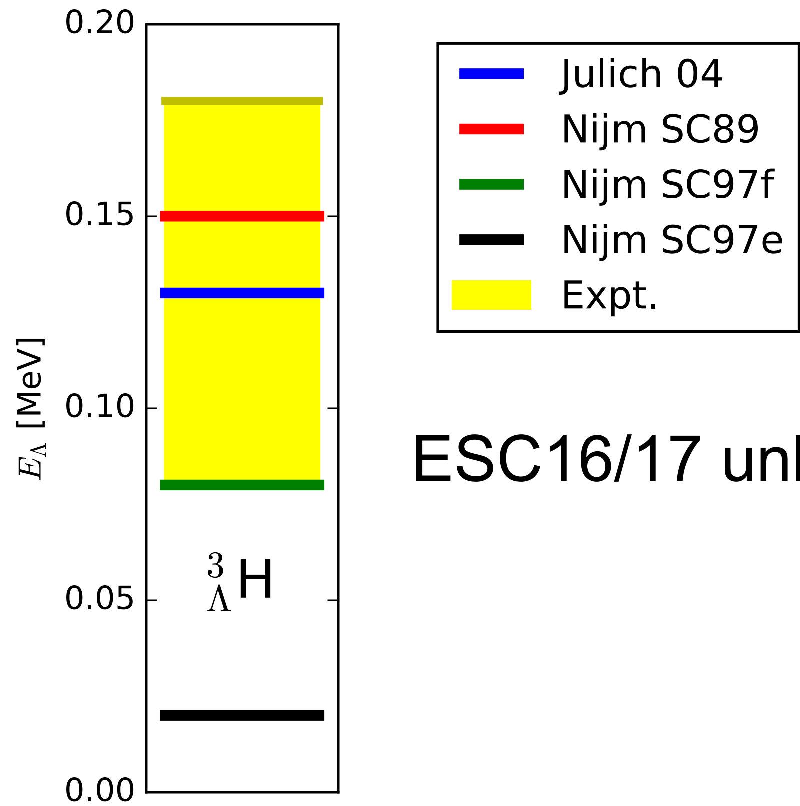
But the direct connection to a YN interaction is lost!

We use two techniques that work for **light hypernuclei** but are based on a direct solution of the hypernuclear (non-relativistic) Schrödinger equation:

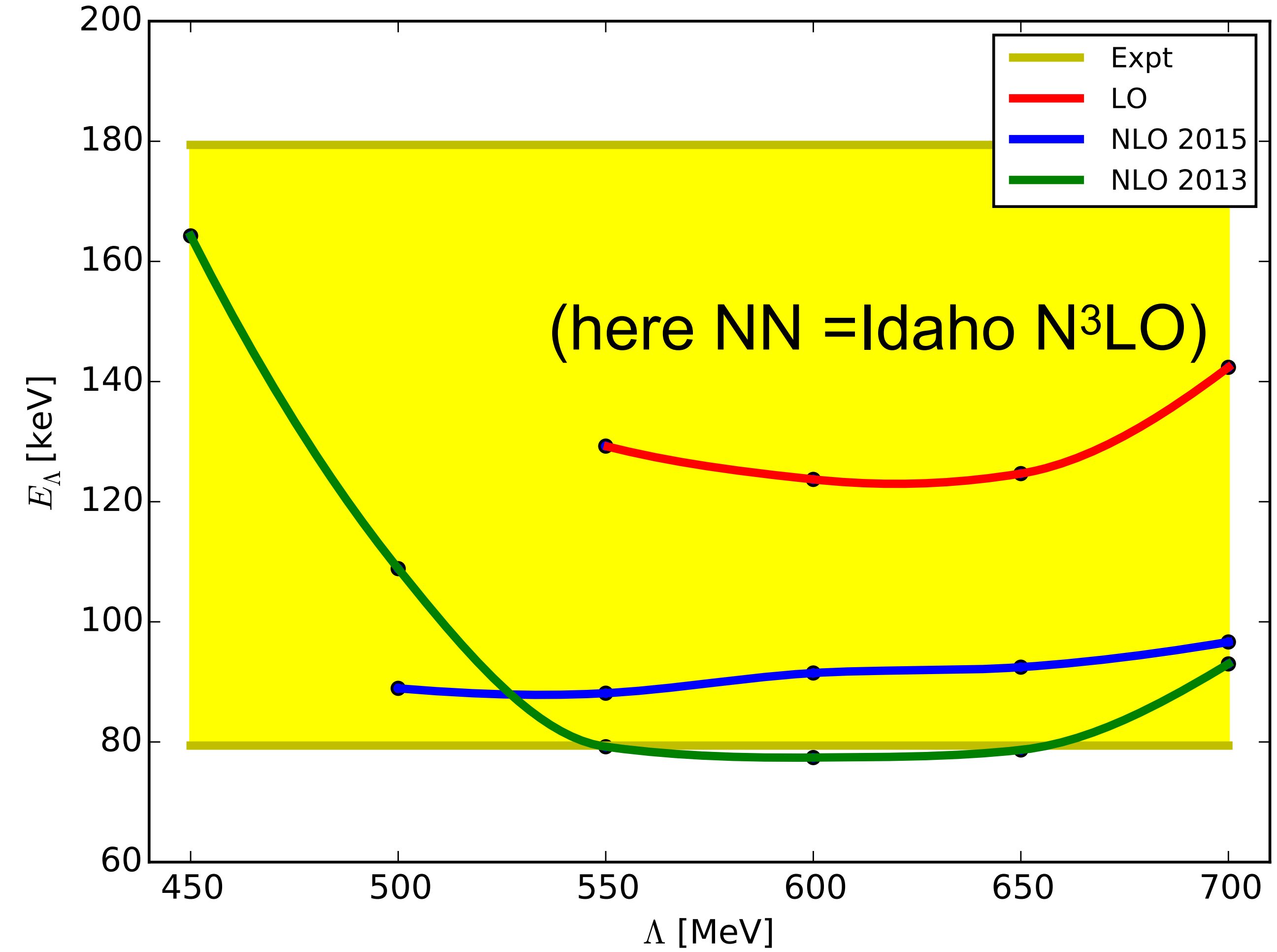
1. solving Faddeev-Yakubovsky eq. in momentum space (used for many years)
2. NCSM using Jacobi HO states as basis (work in progress)

A lot of progress recently: Wirth, Roth, Gazda, Navratil, … (2012-2018)

${}^{3}_{\Lambda}\mathrm{H}$ for chiral & phenomenlogical interactions



ESC16/17 unbound

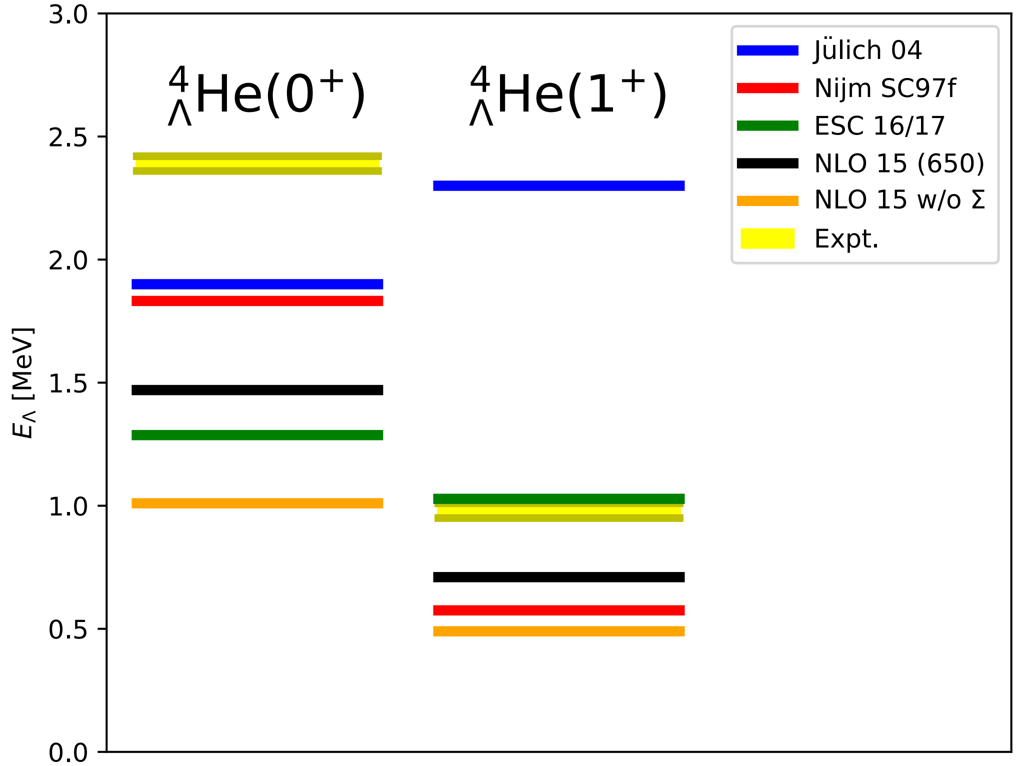


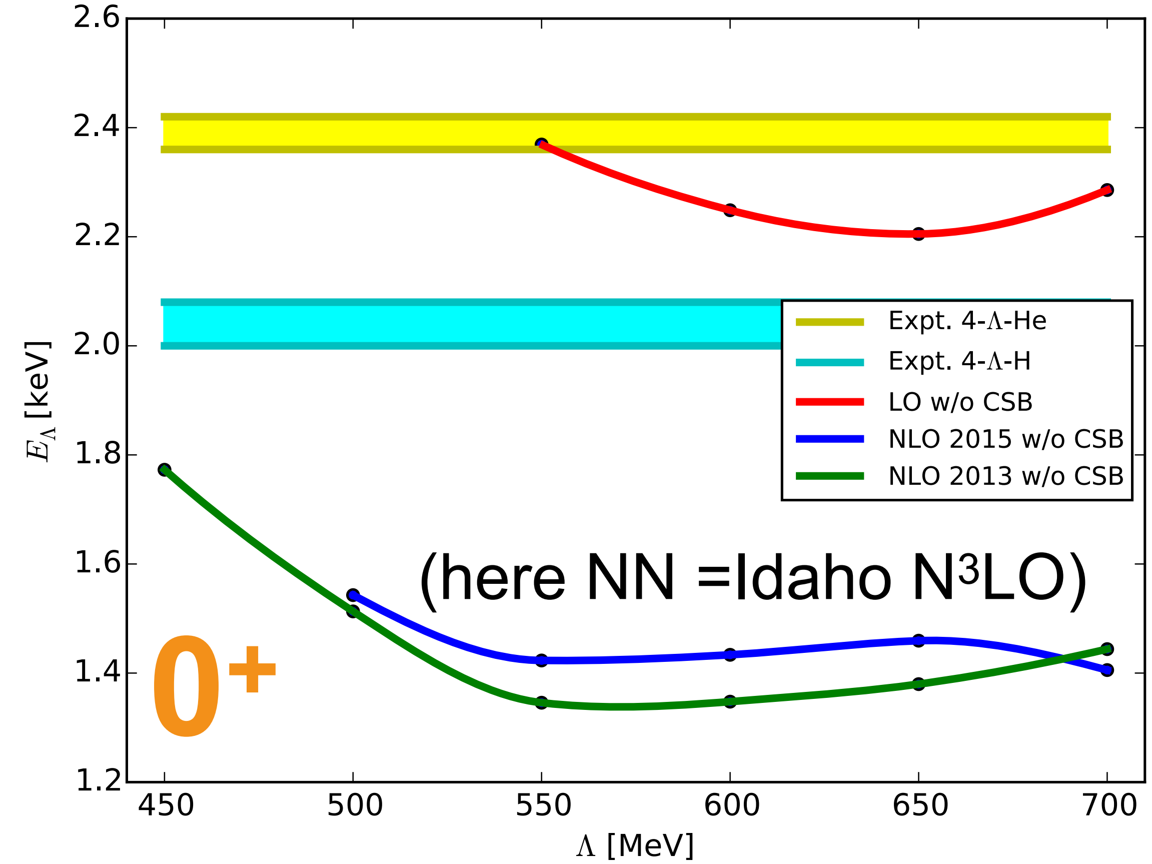
- ${}^{3}_{\Lambda}\mathrm{H}$ is often used to fix relative strength of $^1S_0$ and $^3S_1$ scattering length

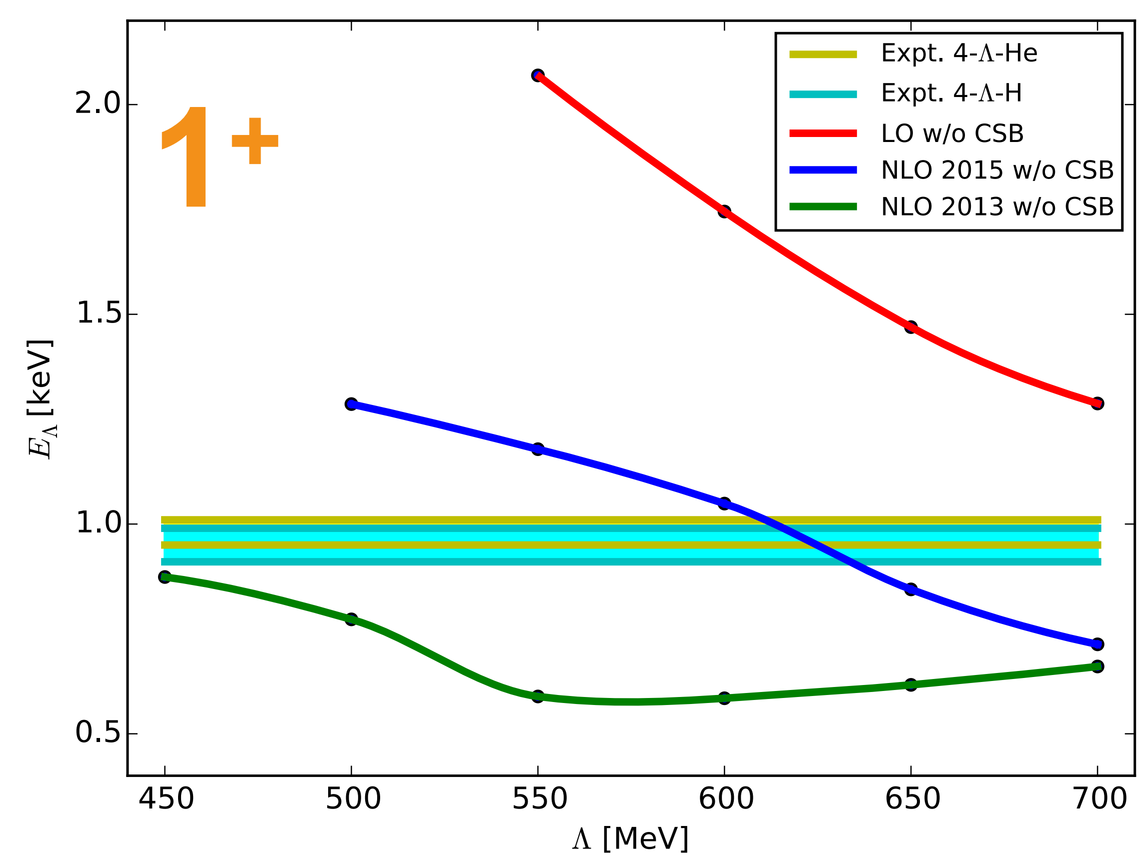
- cutoff variation for chiral interactions
  - is **lower bound** for magnitude of higher order contributions
  - less cutoff dependence for NLO(15)
- two-parameterizations at NLO (2013/2015)

- **3BFs seem to be small**, further insight into (**long range) 3BFs** is needed

$^{4}_{\Lambda}$He for chiral & phenomenlogical interactions







- $^{4}_{\Lambda}$He is **not** well described by any model or LO/NLO interactions
- cutoff variation for chiral interactions is no good estimate of uncertainty in LO
- two-parameterizations at NLO (2013/2015) are similar for the 0$^+$ state but deviate for 1$^+$
- Λ-Σ conversion is related to spin dependence of separation energy

Jacobi-NCSM

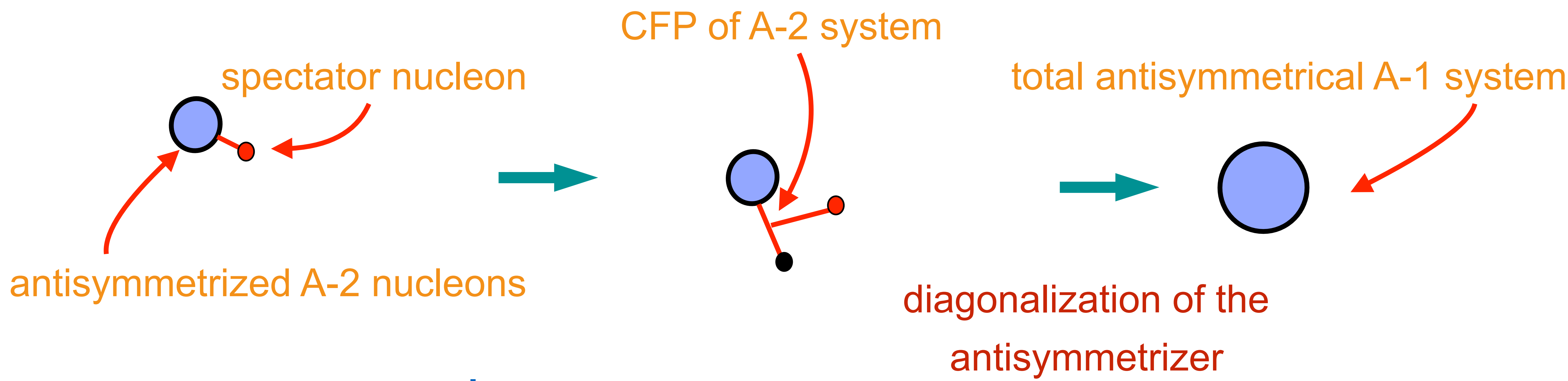
- uses Jacobi coordinates separating off the CM motion
- allows one to go beyond A=4
- efficient for soft interactions
- long distance tails of wave functions cannot be well represented
- requires soft interactions (effective NCSM, vlowk, **SRG**)

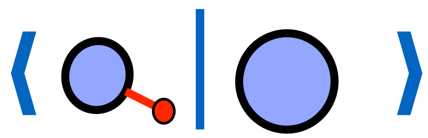
Basic idea: use HO states and soft interactions

- m-scheme uses single particle states (CM not separated)
  antisymmetrization for nucleons easily perform
  larger dimensions (see application to *p*-shell hypernuclei by Wirth et al. (2014,2016))

- Jacobi-NCSM uses relative coordinates
  antisymmetrization for nucleons difficult but possible for $A \leq 8$ (cfp-coefficients)
  small dimensions (see also application to s-shell hypernuclei by Gazda et al. (2014))

First, generate **antisymmetrized states** for the A-1 nucleon system



The CFP coefficients  are obtained by diagonalization of the antisymmetrizer.

HO states guarantee:

- complete separation of antisymmetrized and other states
- independence of HO length/frequency

CFP coefficients will be openly accessible as **HDF5** data files

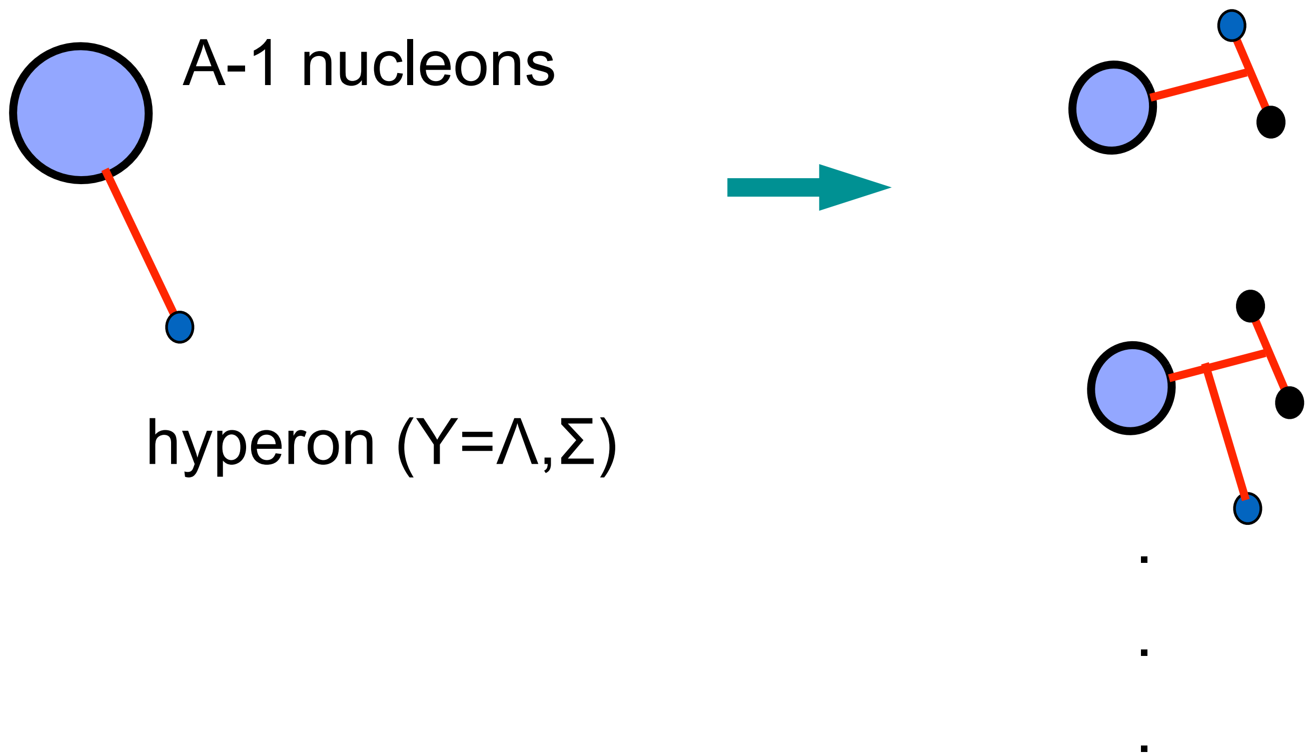
(download server is in preparation *(please ask me when interested!)*)

(Liebig, Meißner, AN (2016))

Jacobi-NCSM

Second, generate **A-body hypernuclei state** (no antisymmetrization required)

Third, rearrange baryons for the application of interactions, …



Again HO states guarantee the independence of HO length/frequency.

Transition coefficients will also be openly accessible as **HDF5** data files

Leads to converged results for "soft" interactions.

(Liebig, Meißner, AN (2016))

(Le, Liebig, Meißner, AN (in progress))

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

$$\frac{dH_s}{ds} = \Big[\underbrace{[T \, , \, H(s)]}_{\equiv \eta(s)} , H(s) \Big] \qquad H(s) = T + V(s)$$

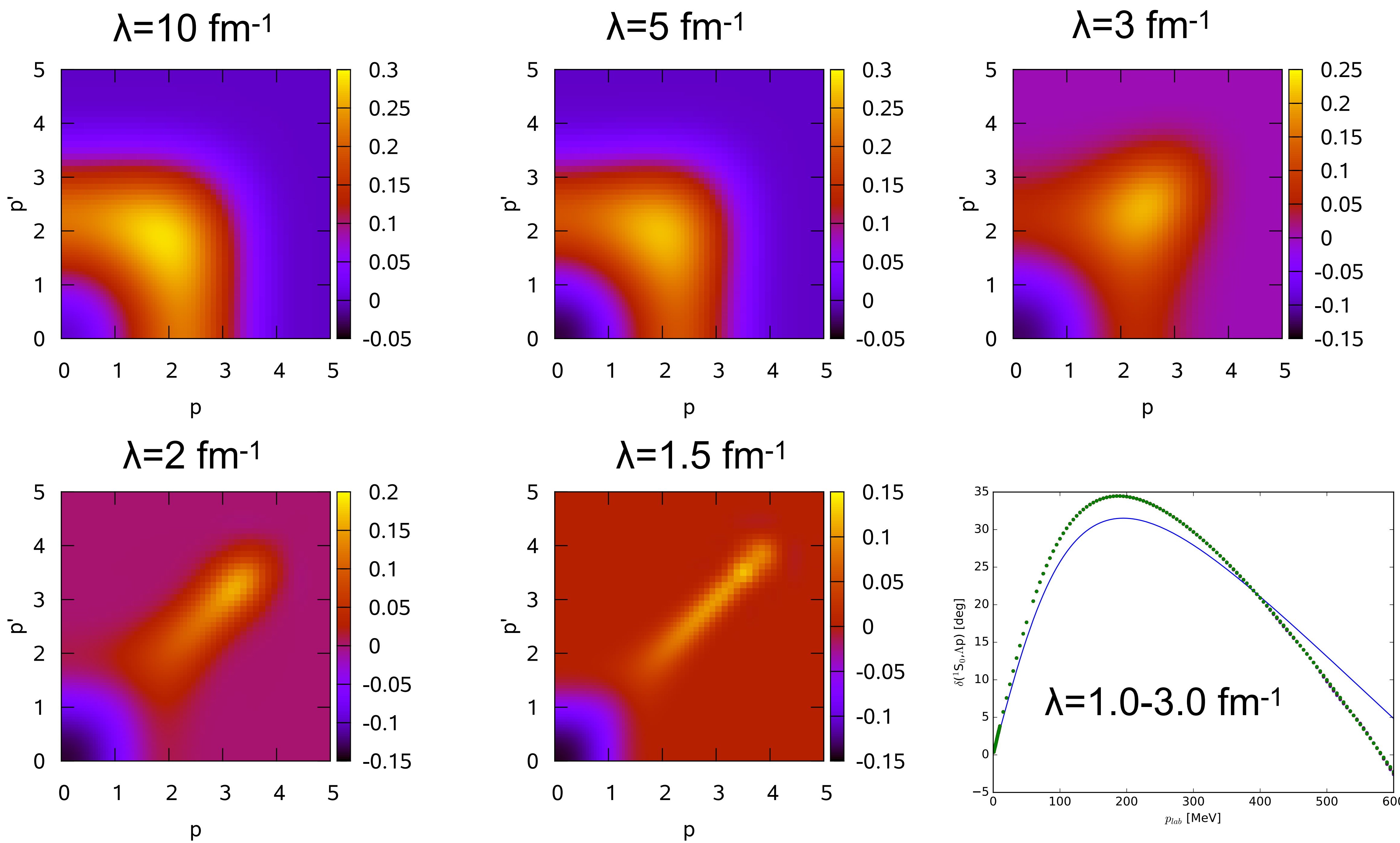
this choice of generator drives *V(s)* into a diagonal form in momentum space

- *V(s)* will be phase equivalent to original interaction
- short range *V(s)* will change towards softer interactions
- 3BF, 4BF, … can in principle be generated but are omitted here
- $\lambda = \left(\frac{4\mu_{BN}^2}{s} \right)^{1/4}$ is a measure of the width of the interaction in momentum space

(Bogner, Furnstahl, Perry (2007))

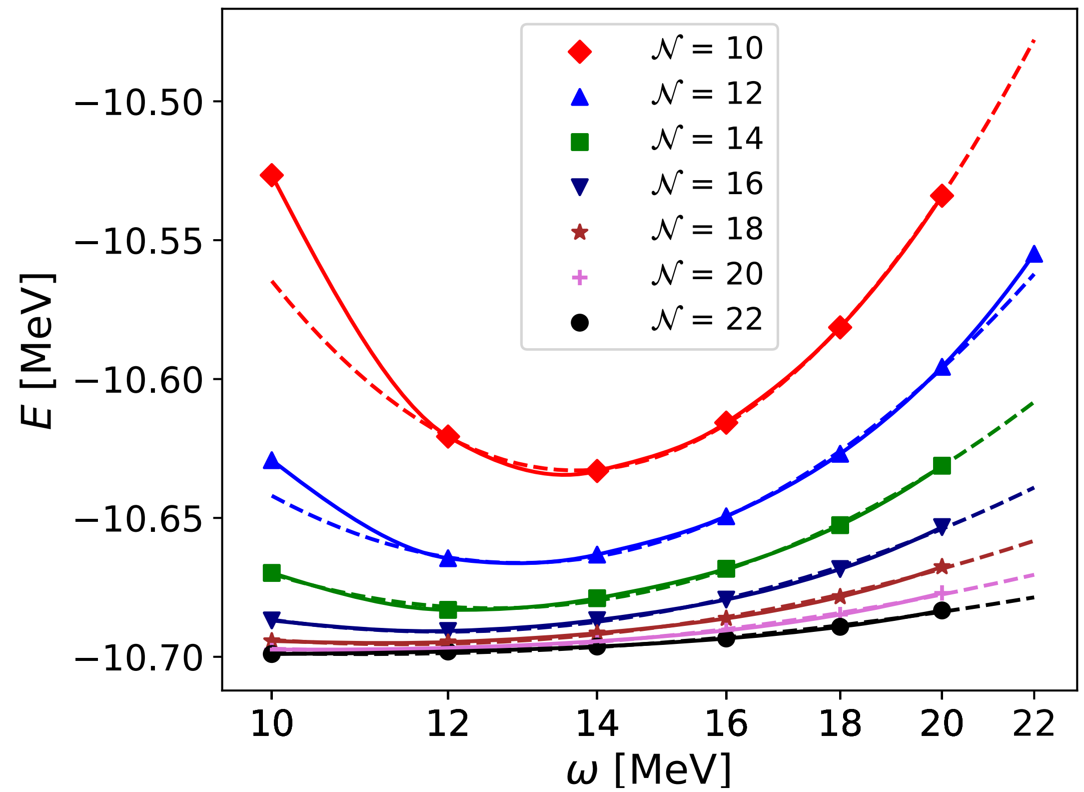
SRG interactions (YN)

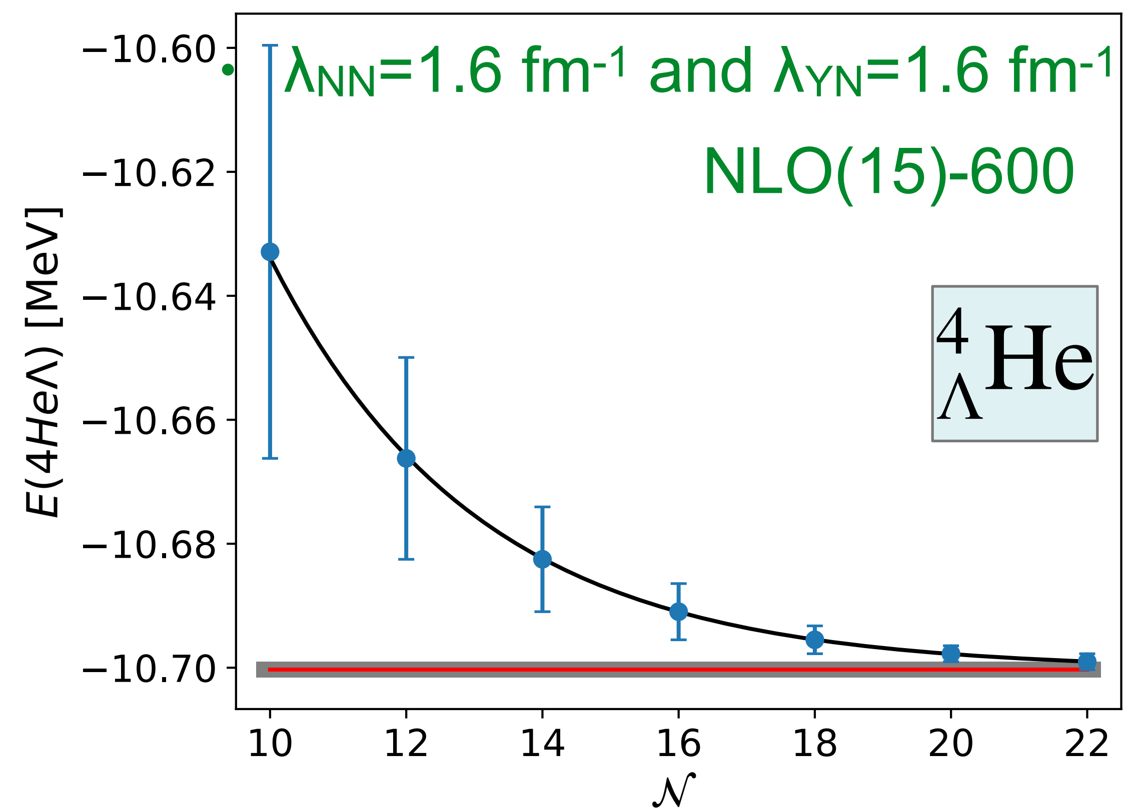
Λp-Λp matrix element for the $^1S_0$ depending on incoming and outgoing momenta



SC97f compared to SRG of EFT-NLO-600

Convergence of NCSM





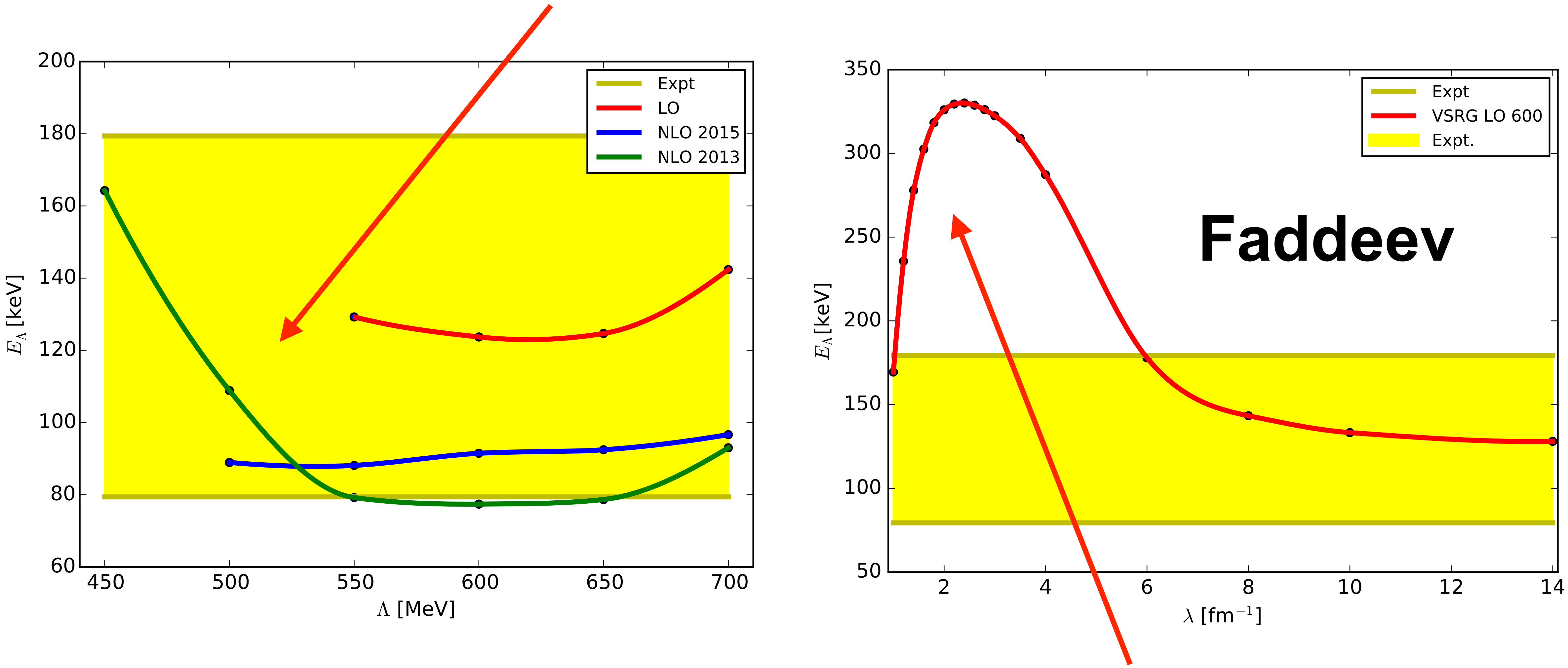
$$E_b(\omega) = E_N + \kappa\left(\log(\omega) - \log(\omega_{opt})\right)^2 \rightarrow E_N = E_\infty + A\,e^{-bN}$$

$$\rightarrow \quad E_\infty = -10.7034(13)$$

- automatized extrapolation of results to converged result
- ω and N dependence is taken into account
- conservative estimate of **numerical** uncertainty

Chiral and SRG 3BF - hypertriton

Cutoff dependence of A=3 separation energy is small for chiral interactions



but: SRG cutoff dependence is much more important than the chiral cutoff dependence!

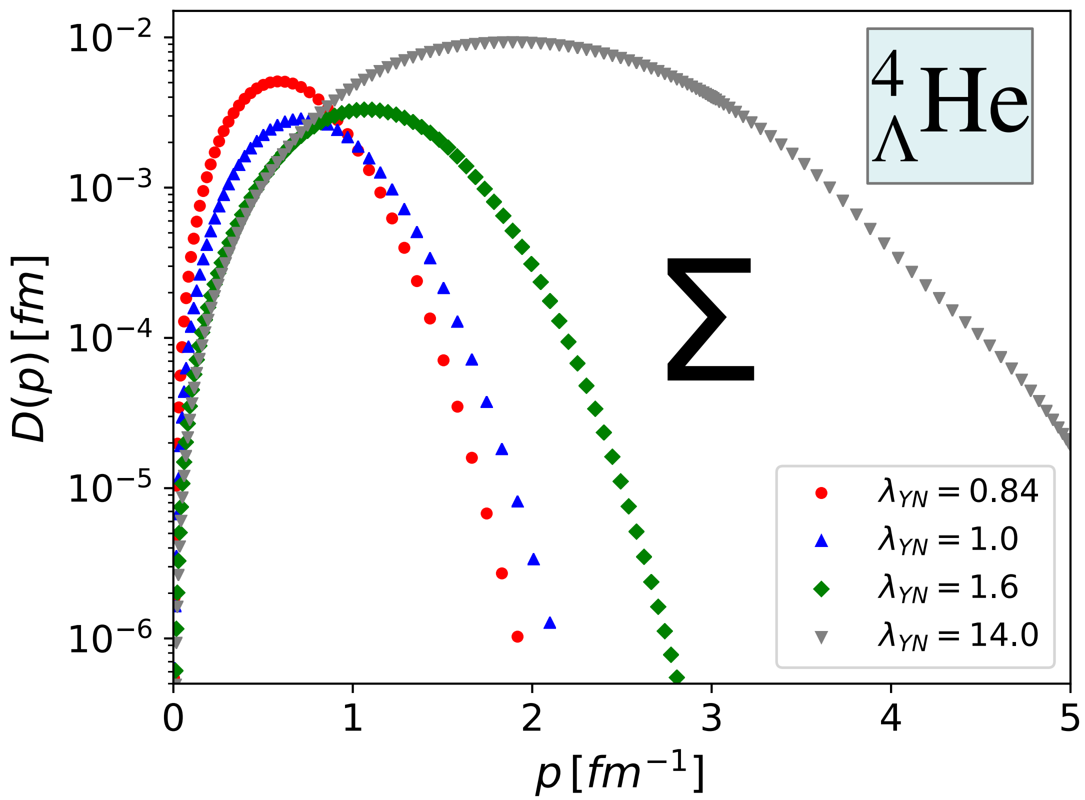
Unfortunately, at least **SRG induced 3BF** are large (see also Wirth, Roth (2016)) !

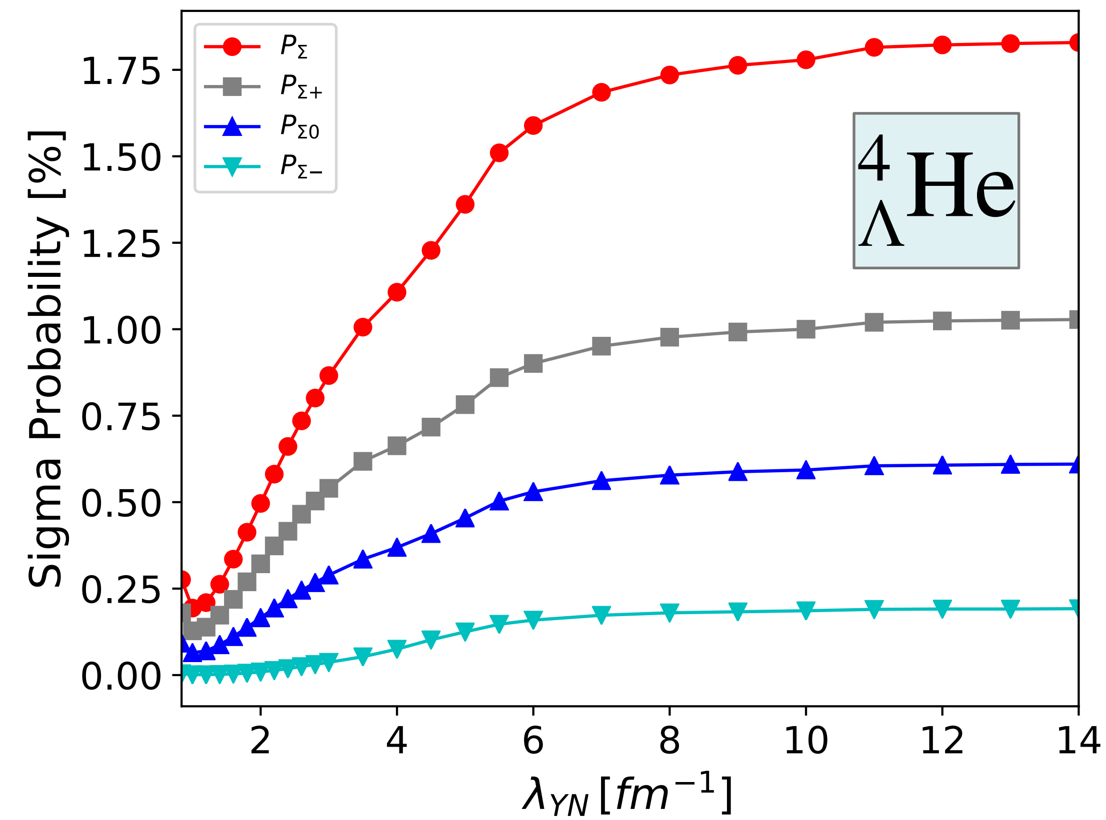
Are parameter-free 3BFs for SRG different to size of chiral 3BFs?

Are 3BFs generally more important for hypernuclei?

Can we learn something from Jacobi-NCSM calculations based on SRG?

Momentum distribution

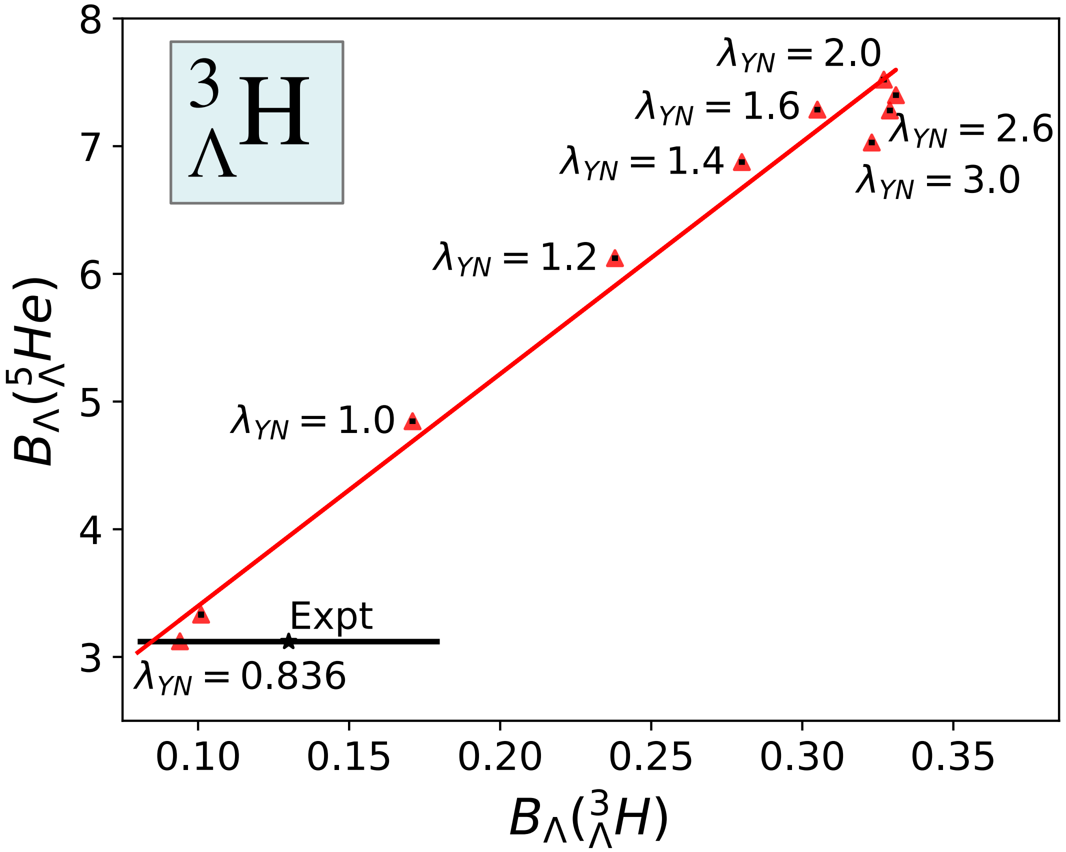


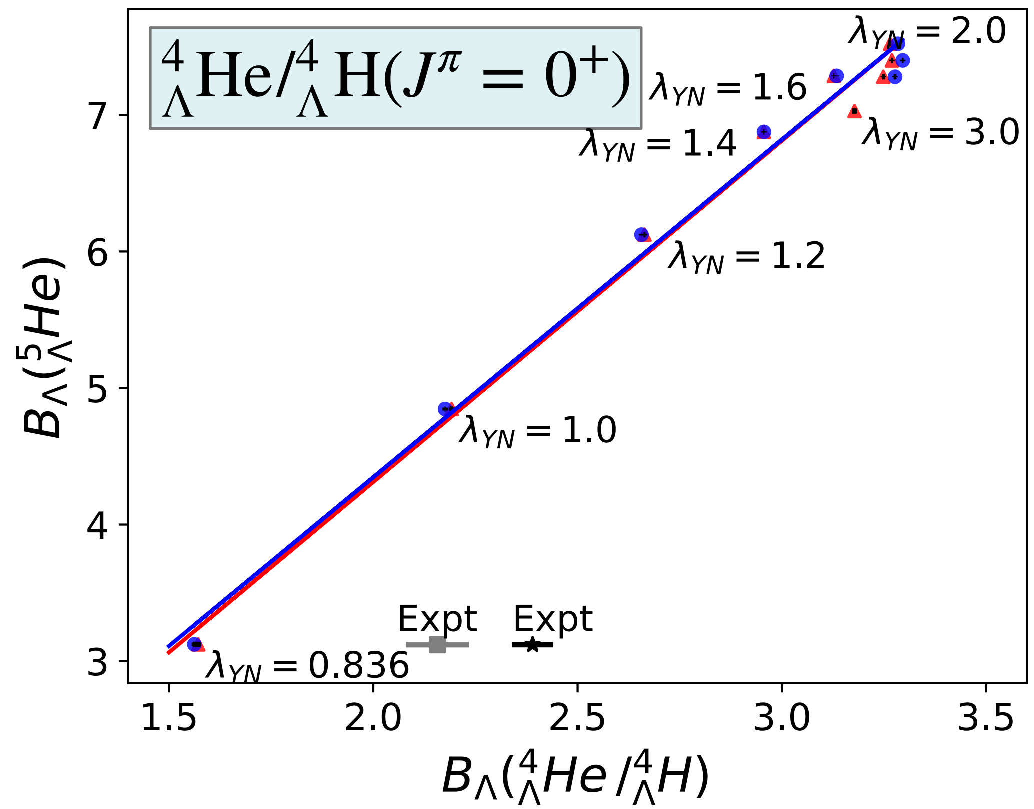


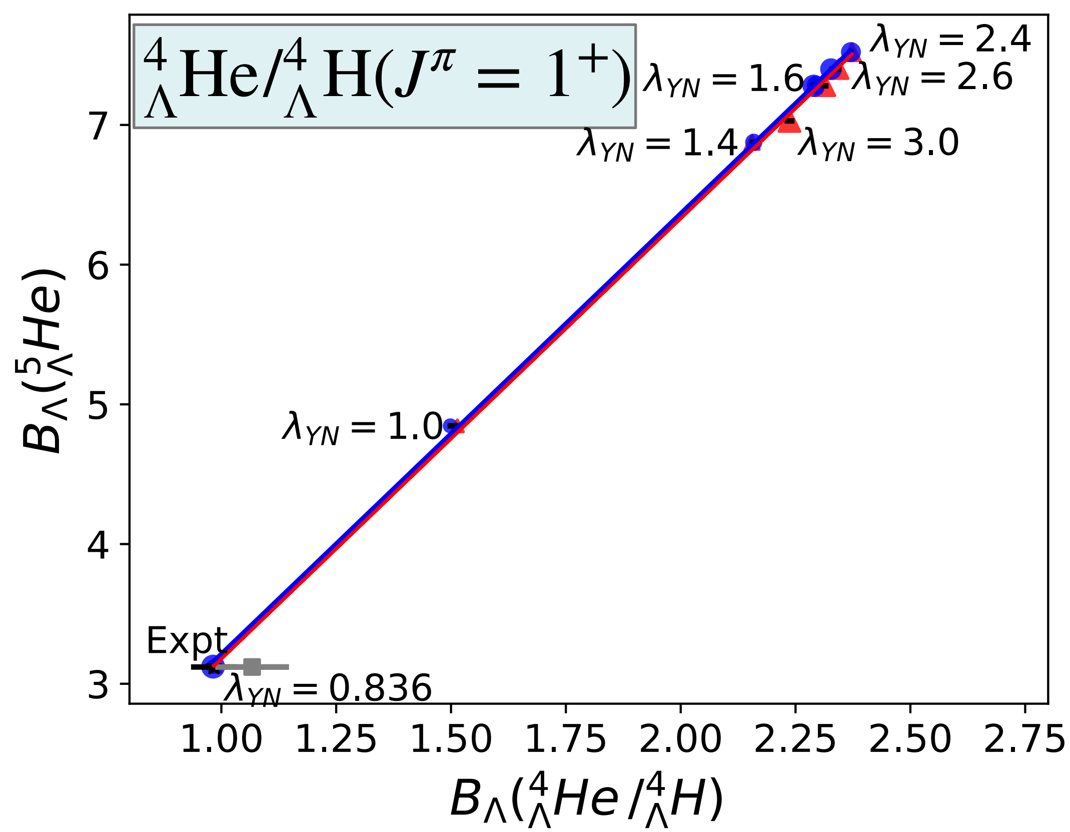
- Σ momentum distribution before SRG has a high momentum tail
- high momentum tail is removed which reduces the Σ probability
- Σ probability is **not** necessarily going to zero !

Correlation of separation energies

Separation energies of s-shell hypernuclei are strongly correlated (to $^{5}_{\Lambda}\mathrm{He}$)



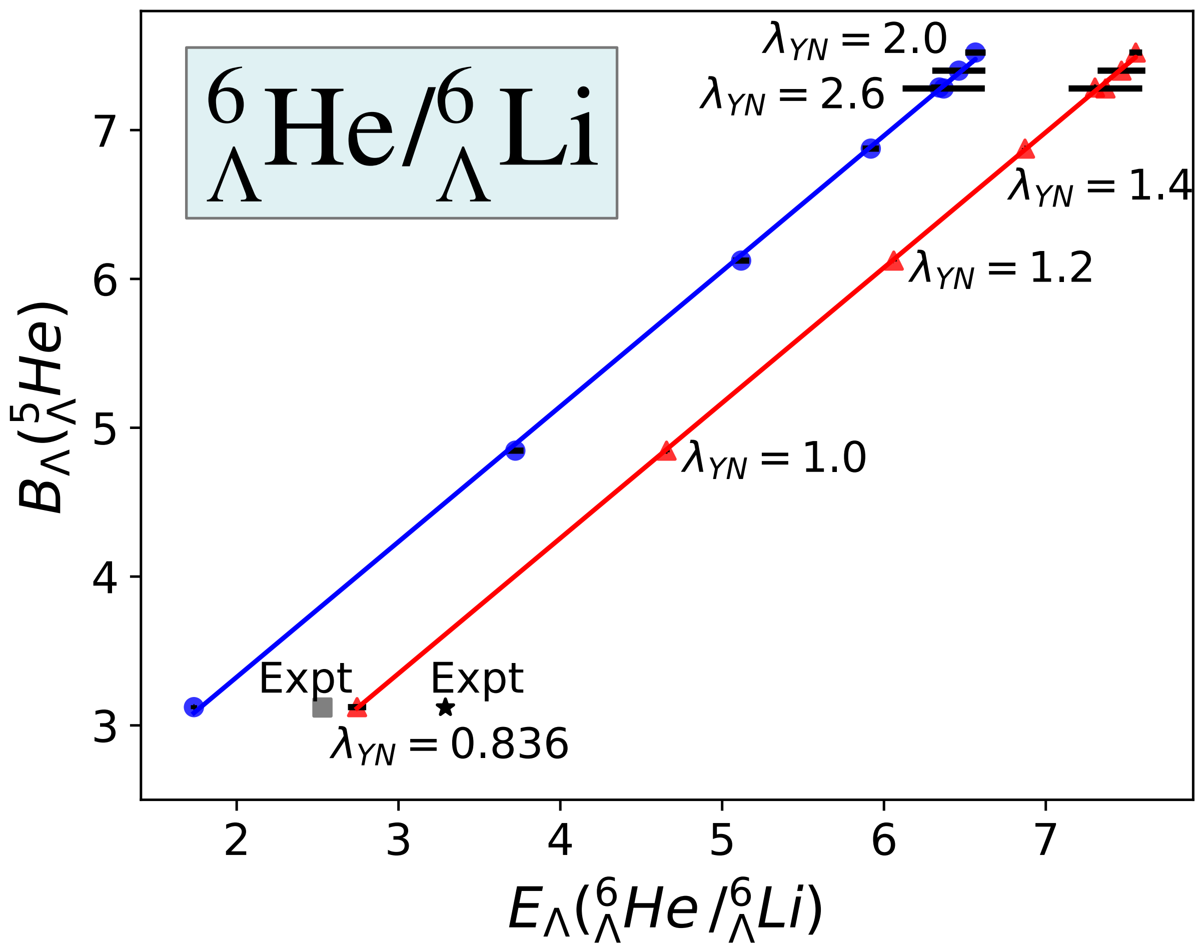


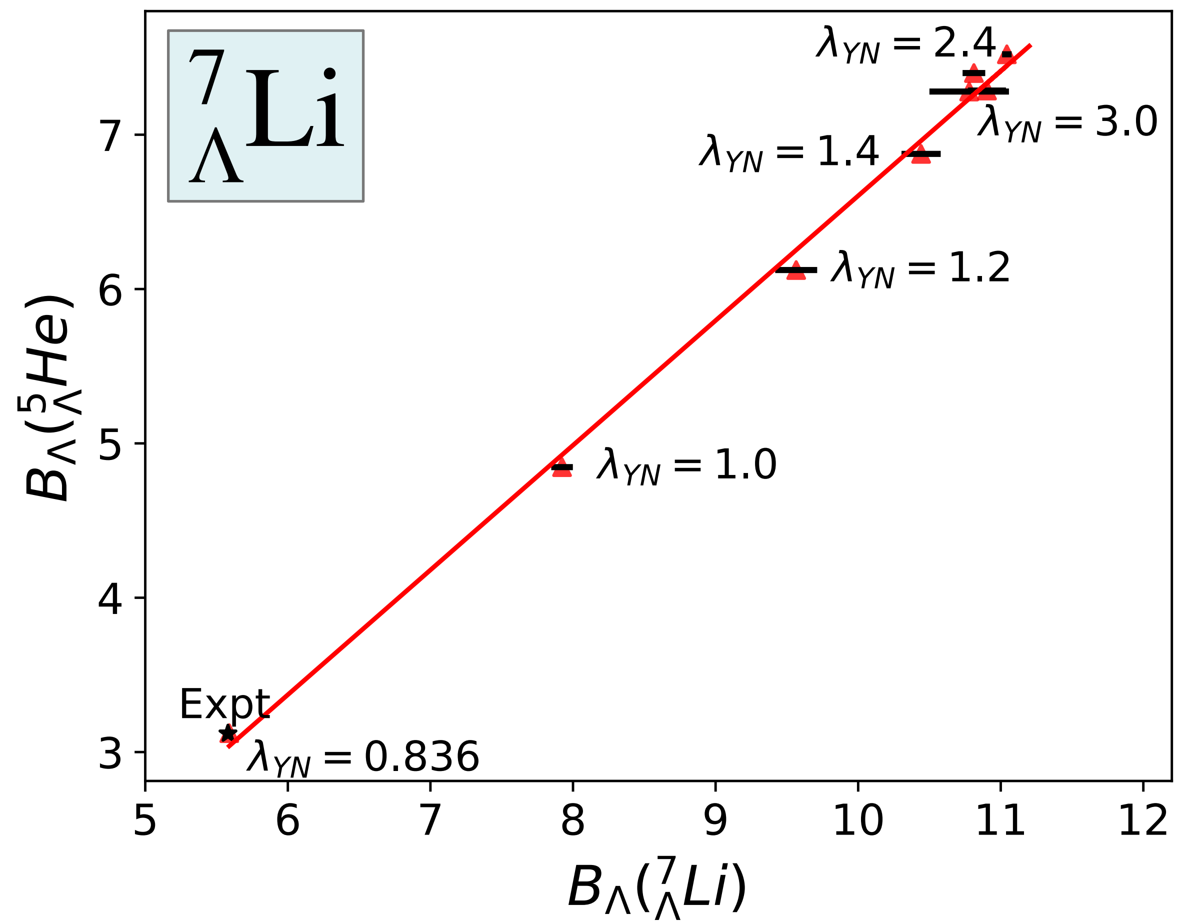


- YN interaction: **NLO(15) 600**
- strong overbinding for $\lambda \gtrsim 1.0\ \mathrm{fm}^{-1}$
- but A=3 and A=5 consistently
  predicted for $\lambda \approx 0.836\ \mathrm{fm}^{-1}$

p-shell hypernuclei

Separation energies of p-shell hypernuclei are also correlated (to $^{5}_{\Lambda}\mathrm{He}$)





- YN interaction: **NLO(15) 600**
- $^{7}_{\Lambda}\mathrm{Li}$ astonishingly well reproduced at "magic" $\lambda \approx 0.836\ \mathrm{fm}^{-1}$
- A=6 in our calculations not particle stable
- NCSM works for narrow resonances
- Coulomb contribution to CSB

Conclusions & Outlook

- YN interactions are interesting and not well understood
  - *Λ-Σ conversion, explicit chiral symmetry breaking*
  - *well known: YN models fail*
  - *NLO of chiral interactions: still freedom to adjust YN forces*

- hypernuclei are an essential source of information on YN forces
  - *it is not trivial to describe the simplest systems consistently*
  - *experiments for **very light** hypernuclei are important!*
    *The data needs to be **accurate** and reliable*
    (better data for the hypertriton or A=4 hypernuclei?)

- Extension of complete calculations to larger systems **(to access more data)**
  - Jacobi-NCSM works and will provide further constraints for YN interactions

- SRG dependence of p-shell results
  - SRG cutoff dependence is large in all systems A=3,4,5, …
  - strong correlations of binding energy can help to avoid SRG-YNN forces
  - we nevertheless **need SRG induced 3BFs** (see also Wirth et al. (2016))
  - further estimates of **3BFs** are needed (implementing Petschauer et al., (2016))