Nucleon-deuteron scattering with chiral semilocal coordinate-space and momentum-space regularized interactions

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Nuclear Theory
in the Supercomputing Era – 2018

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Outline

Introduction:

- Chiral forces AD.2018
- Faddeev approach to 3N scattering

Results:

Nd elastic scattering – predictions and uncertainties

- **Chiral N4LO:** predictions and uncertainties (dependence on regulators, truncation errors, statistical errors)
- **Chiral N4LO:** Ay puzzle

Deuteron breakup - examples

- **Chiral N4LO:** predictions and uncertainties

Summary

Nuclear forces from χEFT – AD 2018

- Big progress in deriving models of NN interacton has been done in last few years (higher orders, new regularization methods, improved LECs fixing, …)

Currently, the most advanced models, applied to Nd scattering, come from:

- the Bochum-Bonn group (E.Epelbaum, U.-G.Meißner, H.Krebs, P.Reinert) 2N (up to N^4LO+), 3N (up to N^3LO) and 4N forces (up to N^3LO)
  - a) SCS (semilocal coordinate space) regularization
    - E.Epelbaum, H.Krebs, U.-G.Meißner, Eur. Phys. J. A51 (2015) 3,26 – up to N3LO
    - E.Epelbaum, H.Krebs, U.-G.Meißner, Phys. Rev. Lett. 115 (2015) 12, 122301 – up to N4LO
  - b) SMS (semilocal momentum space) regularization
    - P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54, 86 (2018).

- the Moscow(Idaho)-Salamanca group (R.Machleidt, D.R.Entem, Y.Nosyk) N^4LO
  - D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96, 024004 (2017).

Other models (e.g. Daejeon16, $NNLO_{OPT}$) are based on works of both mentioned above groups.

Regularization of chiral models

- Various regulators are used for chiral forces
- E.E., H.K., U.-G.M. use local regularization
  - Local regularization in coordinate space $V_{lr}(\mathbf{r}) \rightarrow V_{lr}(\mathbf{r})f(r)$ with $f(r) \equiv \left(1 - e^{-r^2/R^2}\right)^n$
  - R=0.8–1.2 fm what corresponds to Λ=330-500 MeV
  - n=6
  - Regularized potential is transformed to momentum space (different effects in different partial waves)
  - Difficult to apply on 3NF
  - In momentum space regularization is shifted to the pion propagator

$$1/(l^2 - m_\pi^2) \rightarrow F(l^2)/(l^2 - m_\pi^2) \quad \text{with} \quad F(l^2) = e^{\frac{-(l^2 + m_\pi^2)}{\Lambda^2}}$$

Λ=400-550 MeV $\rightarrow$ R=0.7-1.0 fm

- D.R.E., R.M., Y.N. use nonlocal regularization
  - Nonlocal regularization in momentum space V(**p’**,**p**) → V(**p’**,**p**)f(p’,p) with
  - Λ=450-550 MeV
  - n=2,4,…

$$f(p', p) \equiv \exp\left(-\left(\frac{p'}{\Lambda}\right)^{2n} - \left(\frac{p}{\Lambda}\right)^{2n}\right)$$

Formalism for 3N scattering

- 2N bound state: Schrödinger equation,
- 2N scattering state: Lippmann-Schwinger equation for the t-matrix (interaction + free propagation)

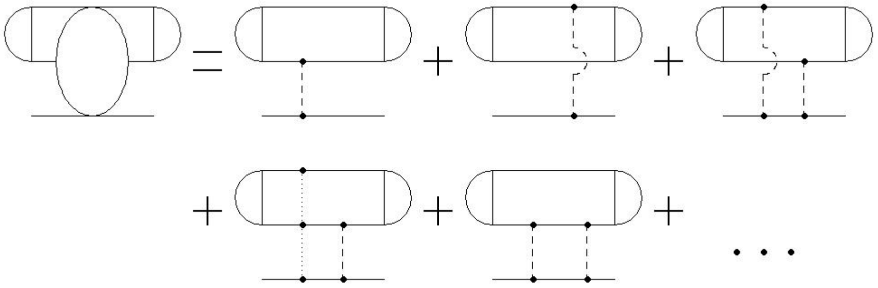
$$t(E) = V + VG_0(E)V + VG_0VG_0(E)V + \dots$$

$$G_0(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E - H_0 + i\varepsilon}$$

- 3N: Faddeev equation

$$T = tP\phi + (1+tG_0)V_{123}^{(1)}(1+P)\phi + tPG_0T + (1+tG_0)V_{123}^{(1)}(1+P)G_0T$$

Transition amplitudes

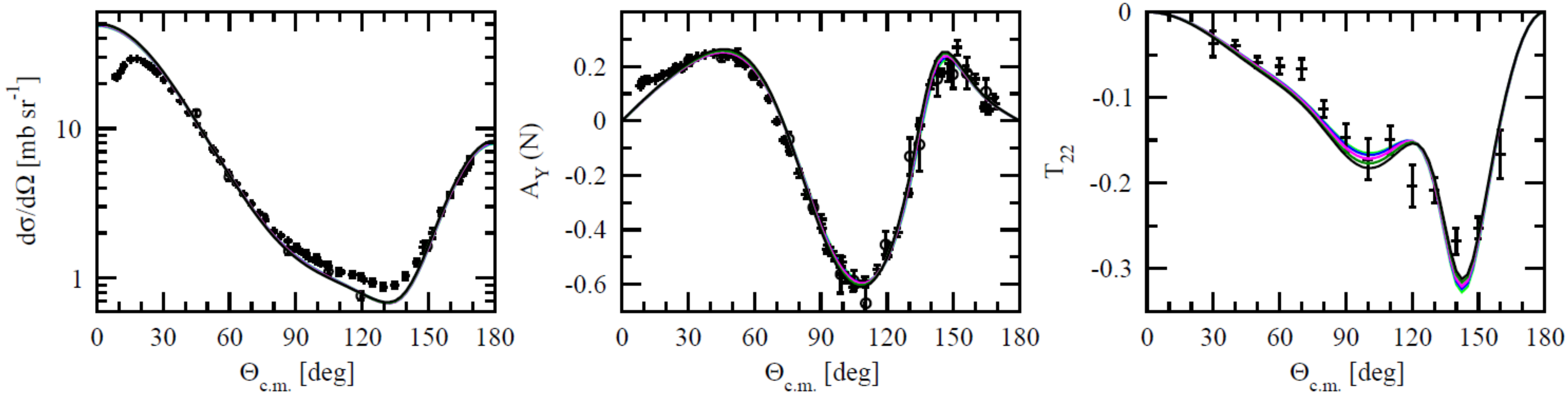


$$U = PG_0^{-1} + V_{123}^{(1)}(1+P)\phi +$$
$$+ PT + V_{123}^{(1)}(1+P)G_0T$$
$$U_0 = (1+P)T$$

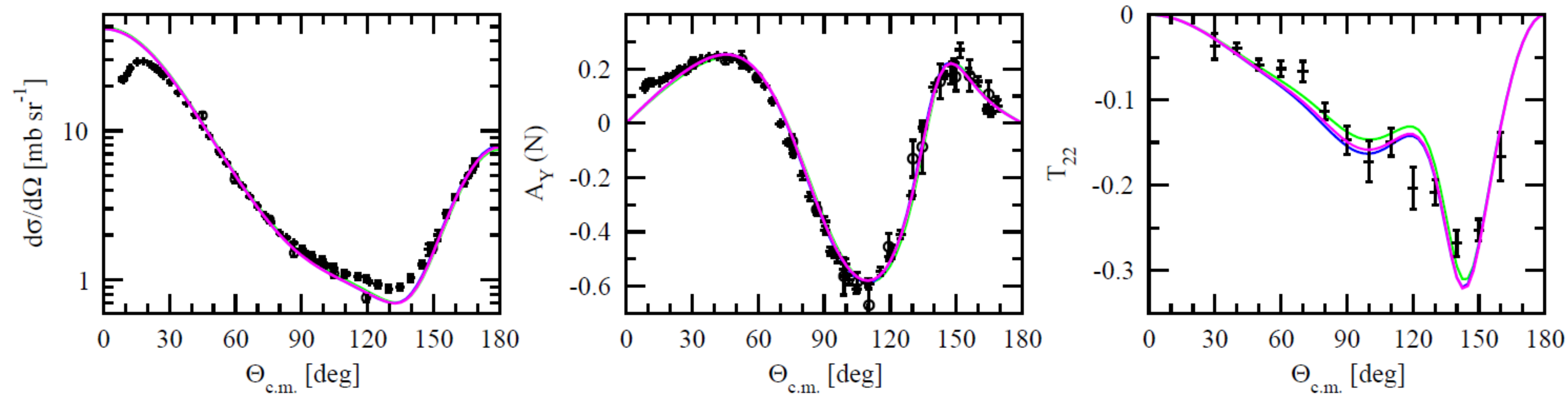
W.Glöckle et al., Phys. Rept. 274, 107 (1996); J.Golak et al., Phys. Rept. 415, 89 (2005)

Test: regulator dependence at 65 MeV

E.E.,H.K.,U.-G.M. (SCS, N^{4}LO, R=0.8-1.2 fm)



D.R.E.,R.M.,Y.N. (N^{4}LO, Λ=450-550 MeV, 2017)



Test: regulator dependence at 65 MeV

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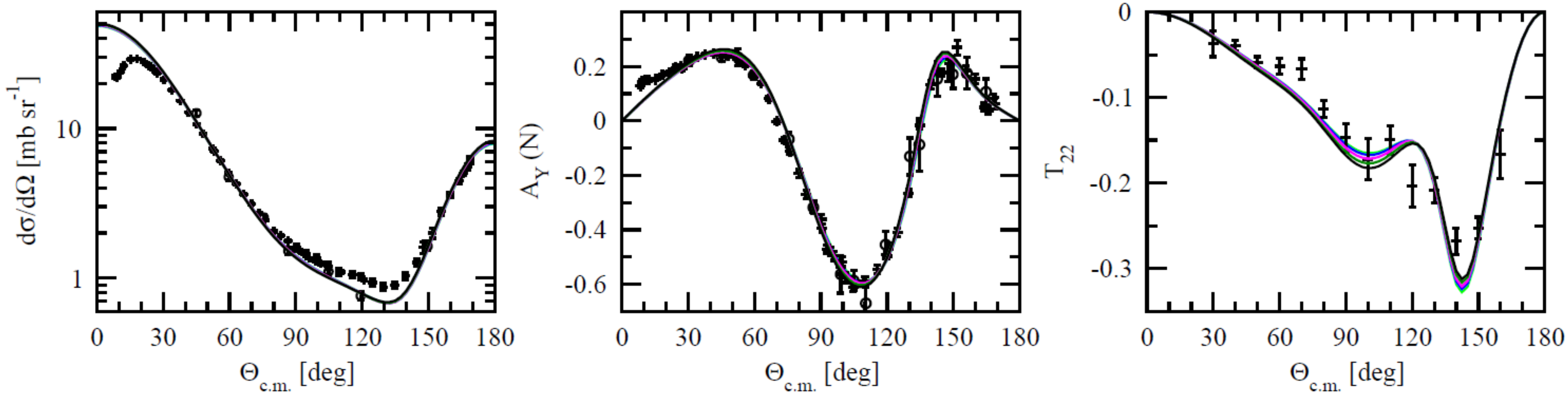
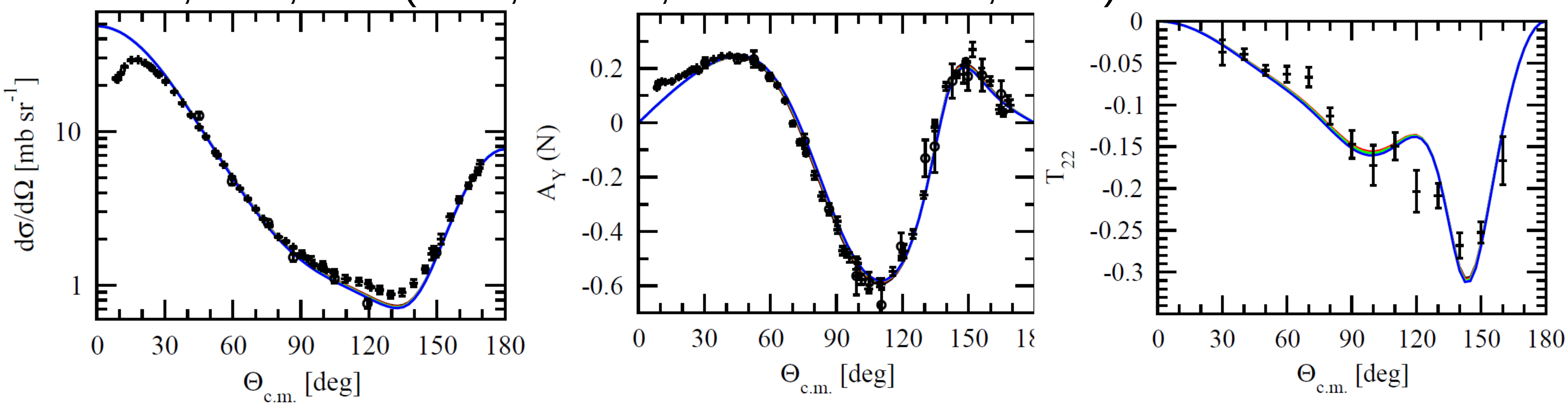


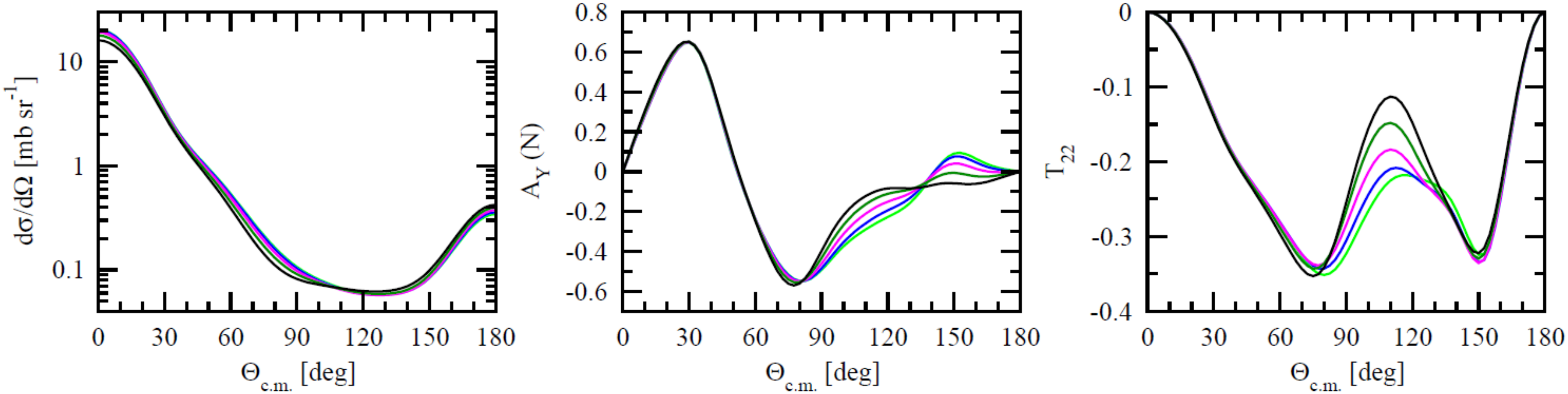
P.R., H.K., E.E. (SMS, N^4LO+, Λ=400-550 MeV, 2018)



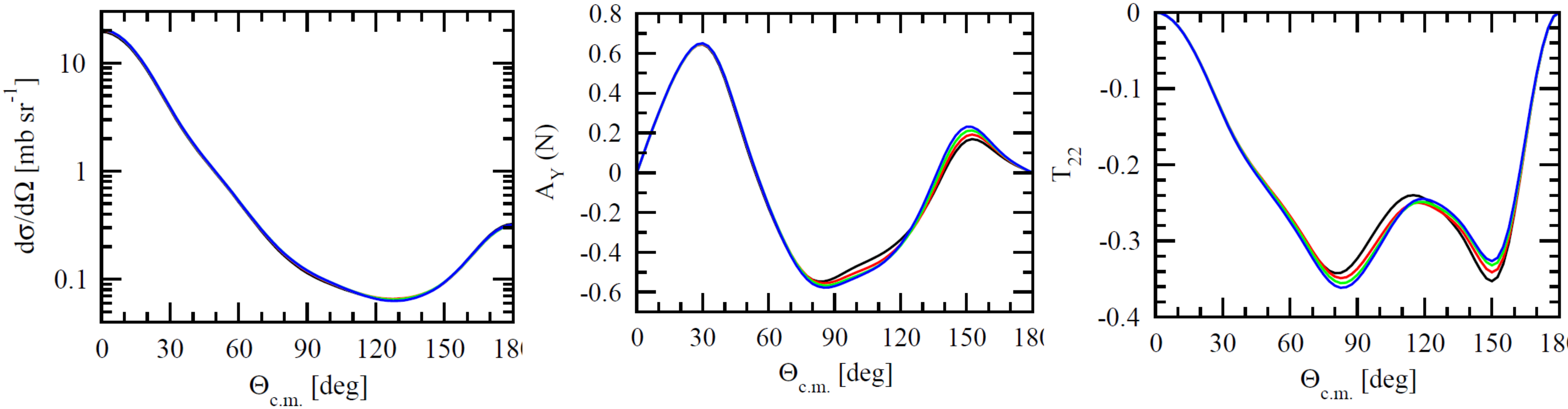
→ both local regularizations lead to very small regulator dependence (at E=65 MeV). Improvement for SMS.

Test: regulator dependence at 200 MeV

E.E.,H.K.,U.-G.M. (SCS N^{4}LO, R=0.8-1.2 fm)



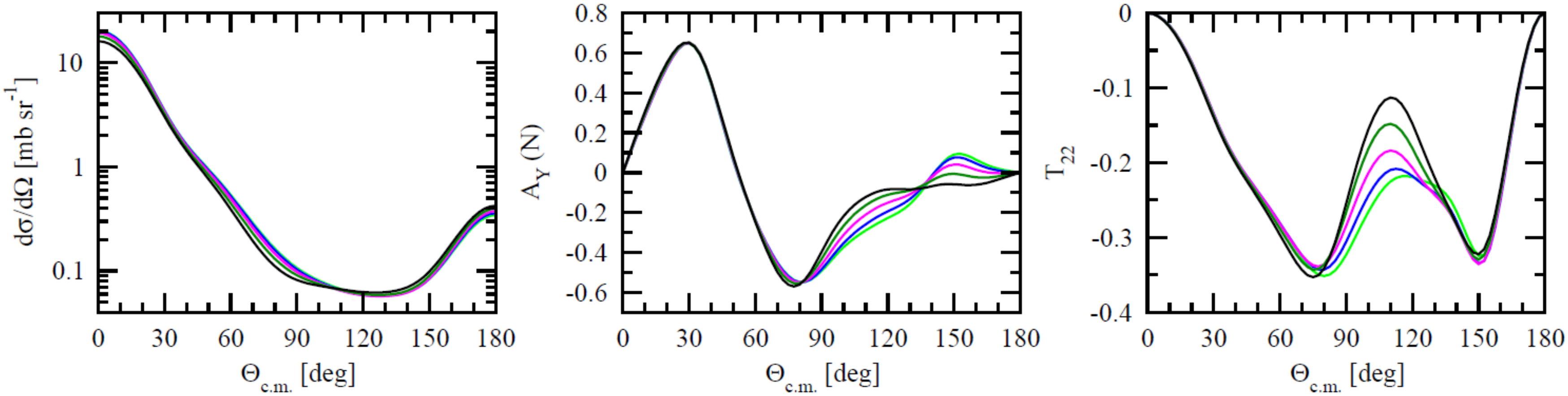
P.R., H.K., E.E. (SMS, N^{4}LO+, Λ=400-550 MeV, 2018)



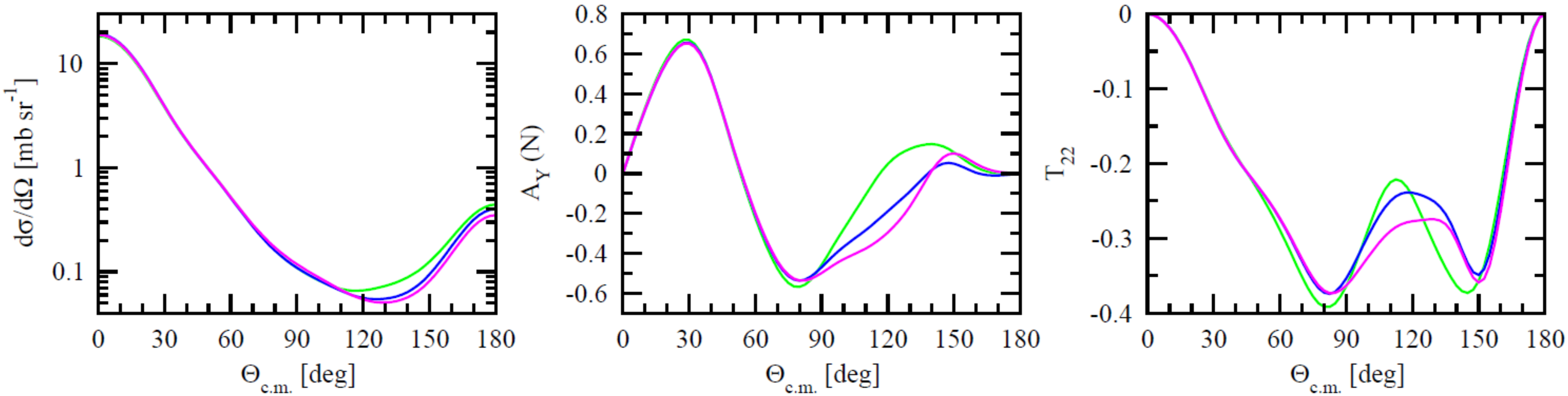
→ substantially smaller regulator dependence for SMS force.

Test: regulator dependence at 200 MeV

E.E.,H.K.,U.-G.M. (SCS N^{4}LO, R=0.8-1.2 fm)



D.R.E.,R.M.,Y.N. (N^{4}LO, Λ=450-550 MeV, 2017)



→ big regulator dependence but energy is at limit of χEFT applicability

Truncation errors

- We use prescription proposed in S.Binder et al., Phys. Rev. C93 (2016) 044002.

$$\Delta X^{(2)} = X^{(2)} - X^{(0)}$$

$$\Delta X^{(i)} = X^{(i)} - X^{(i-1)}$$

$$\delta X^{(0)} = Q^2 \left| X^0 \right|$$

$$\delta X^{(2)} = \max\left(Q^3 \left| X^0 \right|, Q^1 \left| \Delta X^{(2)} \right| \right)$$

$$i \geq 3: \ \delta X^{(i)} = \max\left(Q^{i+1} \left| X^{(0)} \right|, Q^{i-1} \left| \Delta X^{(2)} \right|, Q^{i-2} \left| X^{(3)} \right| \right)$$

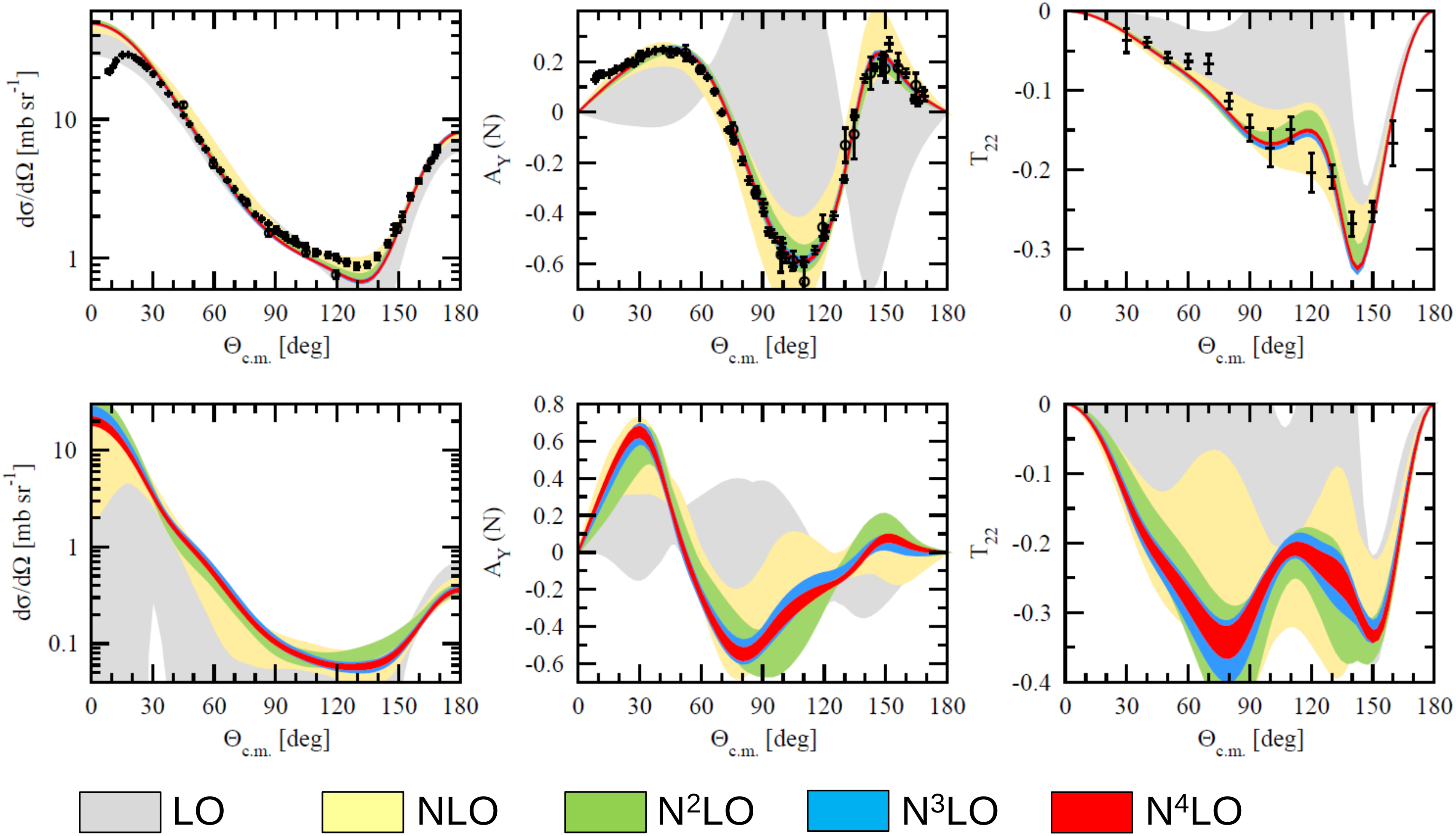
$$Q = \max\left({}^{m_\pi}\!/_{\Lambda_b}, {}^{p}\!/_{\Lambda_b} \right)$$

$$\delta X^{(2)} \geq Q \delta X^{(0)}$$

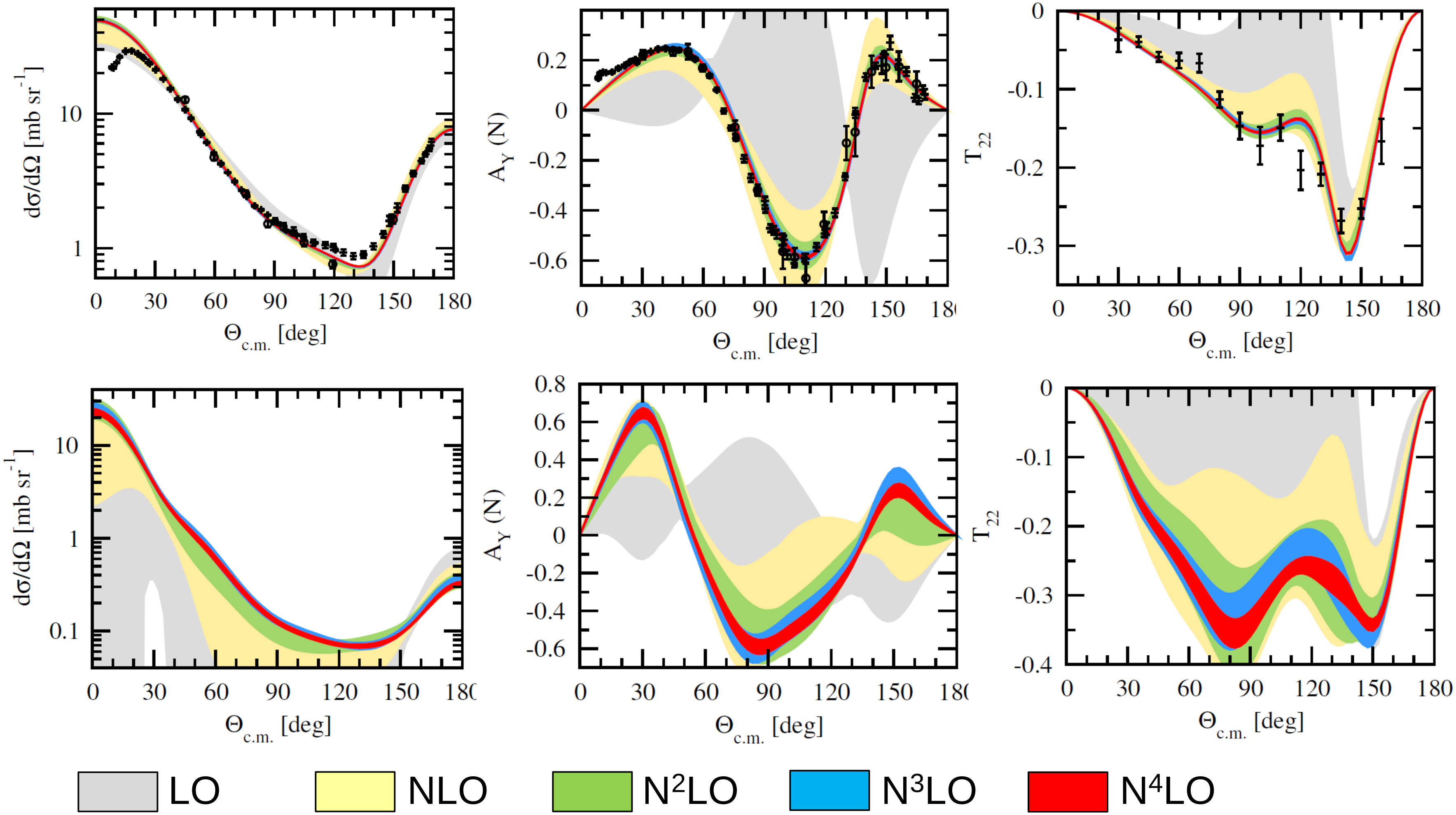
$$\delta X^{(i \geq 3)} \geq Q \delta X^{(i-1)}$$

- This simple prescription is in agreement with more advanced Bayesian analysis discussed in R.J.Furnstahl et al., Phys. Rev. C92 (2015) 024005.
- Note, E.E, H.K., U.-G.M use Λ_b=600 MeV and D.R.E.,R.M.,Y.N. use Λ_b=1000 MeV.

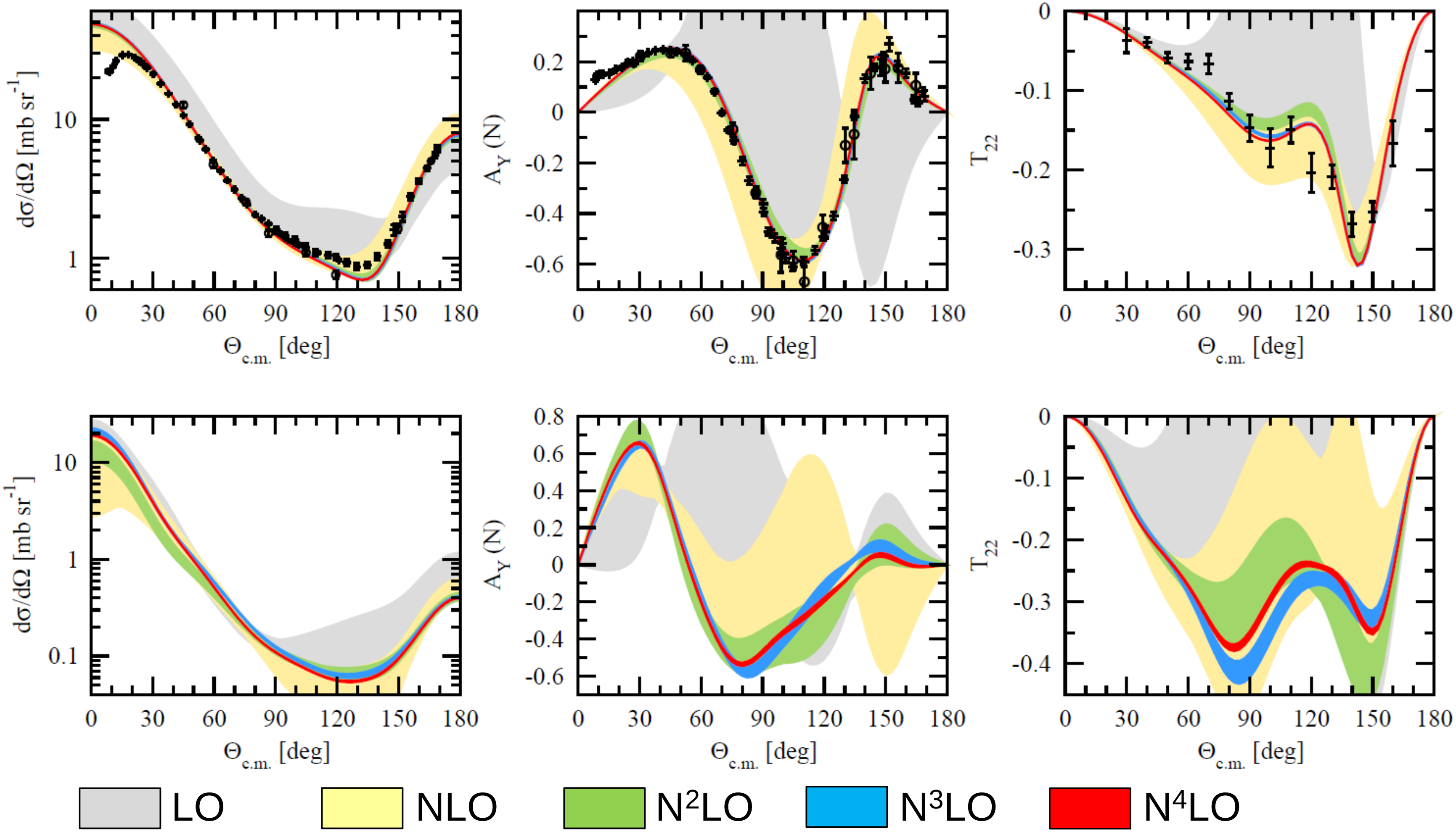
Truncation errors for SCS force at 65 and 200 MeV



Truncation errors for SMS force at 65 and 200 MeV



Truncation errors for D.R.E.,R.M.,Y.N. at 65 and 200 MeV



How to estimate statistical uncertainties ?

- Statistical uncertainties – here uncertainties of 3N observables arising from uncertainties of 2N force parameters

- Knowing 2N force parameters and their correlation matrix we sample many (50) sets of potential parameters

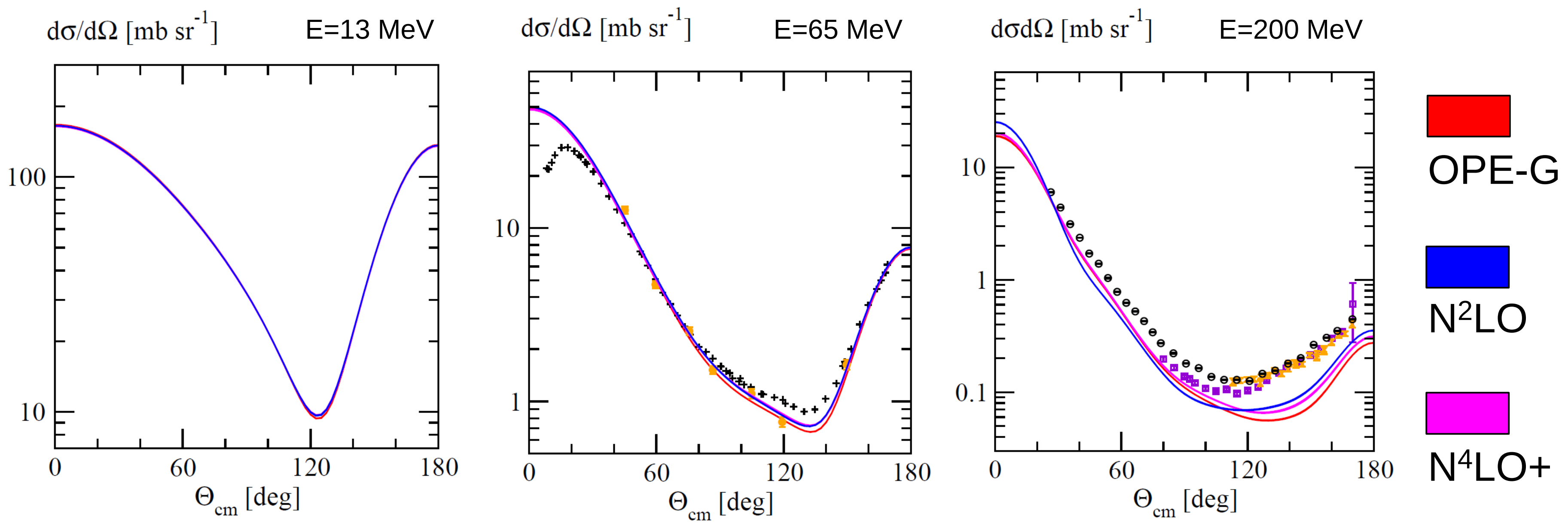
- For each set we solve Faddeev equation and compute 3N observables.
- Thus for each observable (at given energy and scattering angle) we have 50+1 predictions.

- Basing on these predictions we estimate the uncertainty of given 3N observable. This can be done in various ways, which in practice leads to similar results. We use $\Delta_{68\%}$ - a difference between maximal and minimal value of observable taken after neglecting most extreme predictions.

More discussion in R.S. et al., Phys. Rev. C98, 014001 (2018).

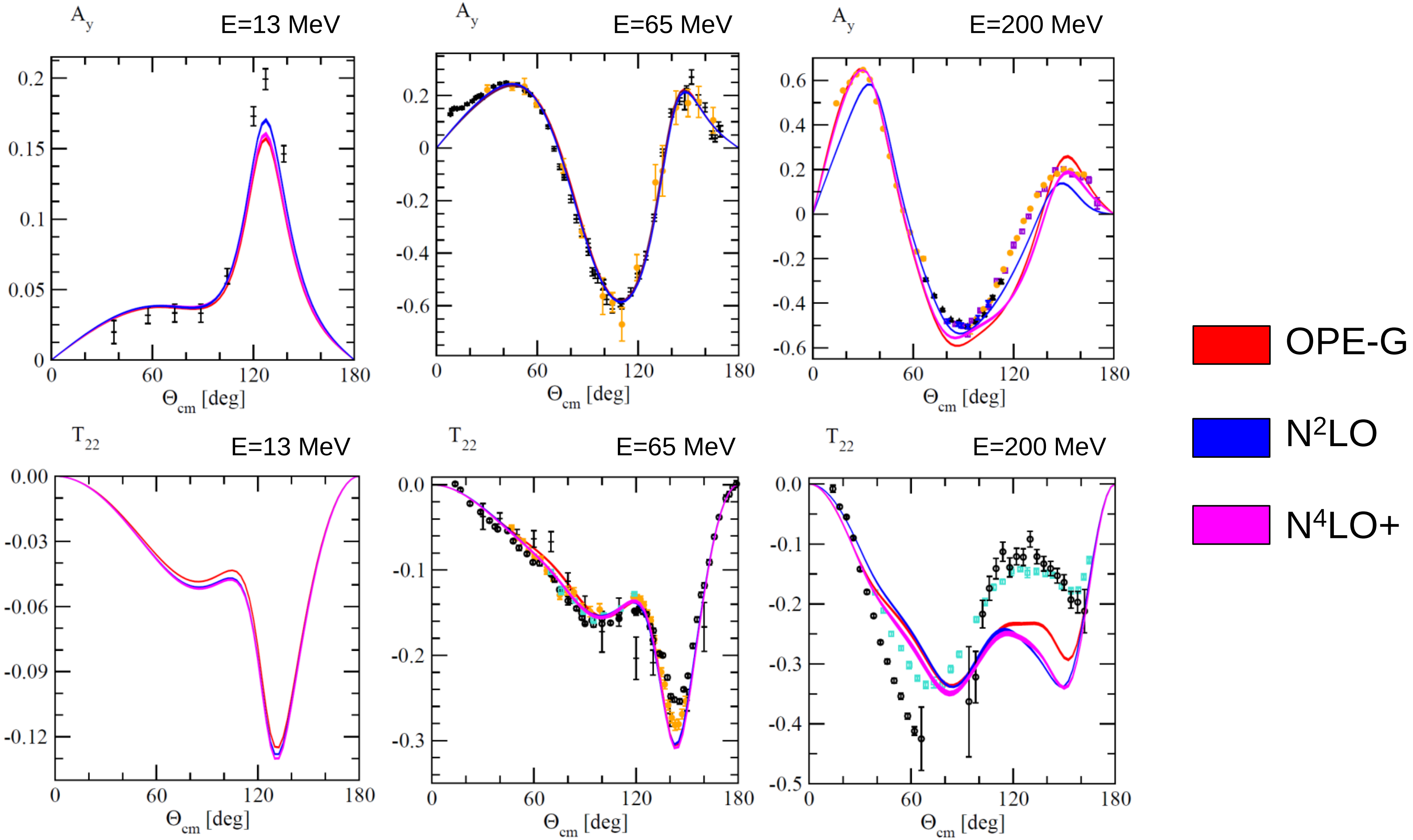
Statistical errors with chiral forces

- New SMS potential also allows to study propagation of uncertainties to 3N system



- Statistical errors are small, also at E=200 MeV.
- Statistical errors for the SMS force are of similar magnitude as the ones for the OPE-Gaussian force (R.Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89 (2014) 064006)
- Similar magnitudes at N^{2}LO and N^{4}LO+.

Statistical errors with SMS chiral forces

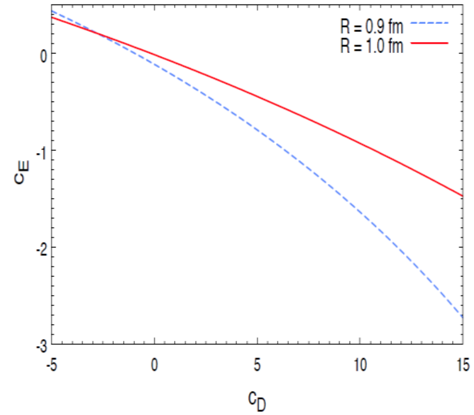


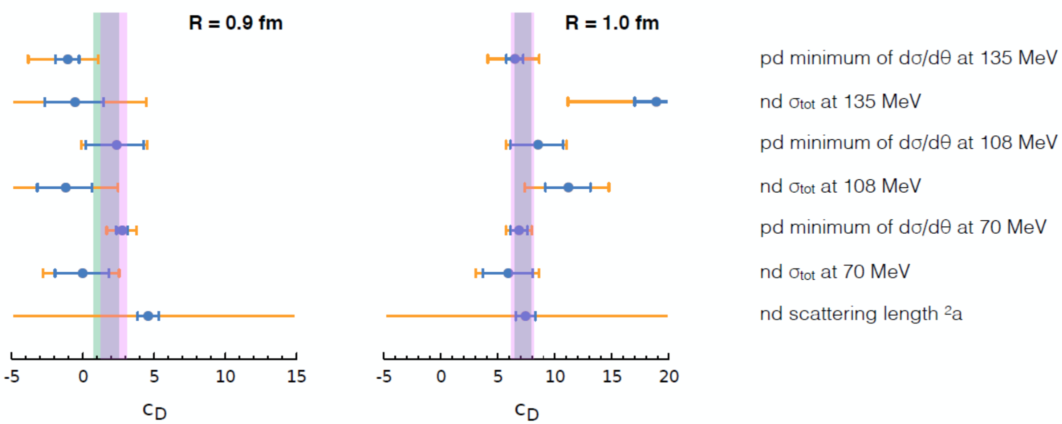
- Statistical errors for the SMS force are of similar magnitude as the ones for the OPE-Gaussian.

Fixing parameters of 3NF

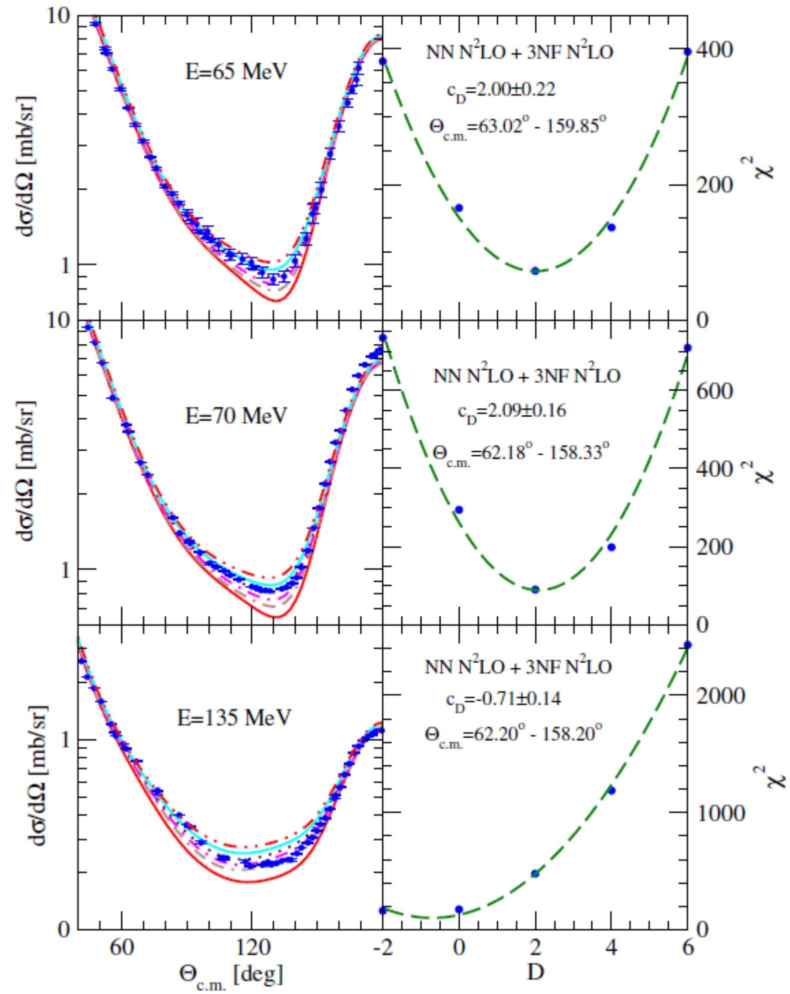
To fix c_D and c_E we use:

- ^{3}H binding energy
- Nd elastic cross section





N2LO NN + N2LO 3NF



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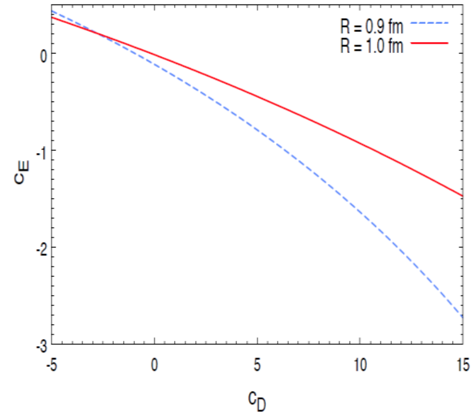
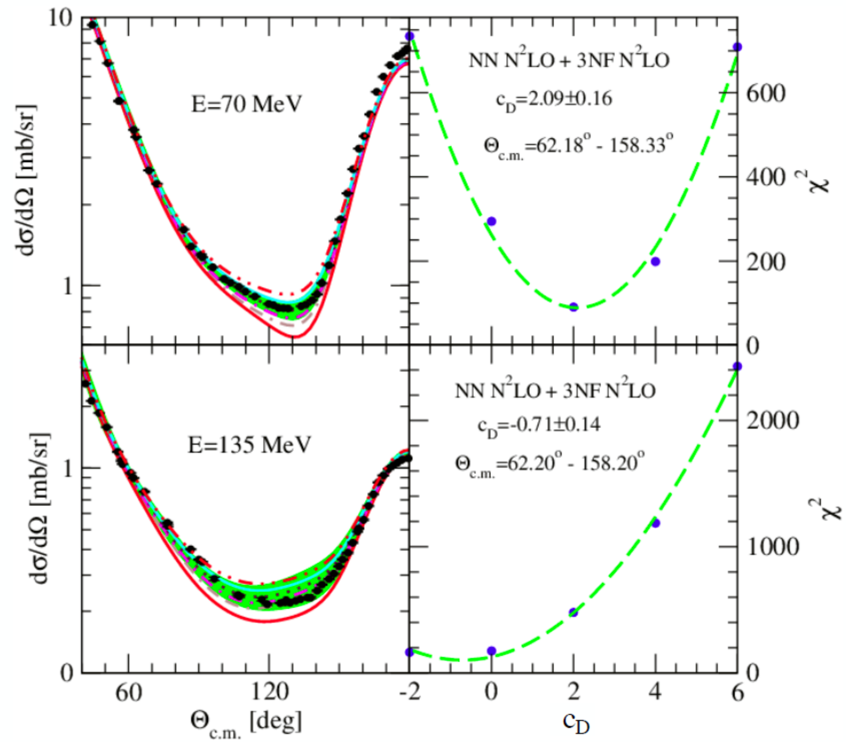
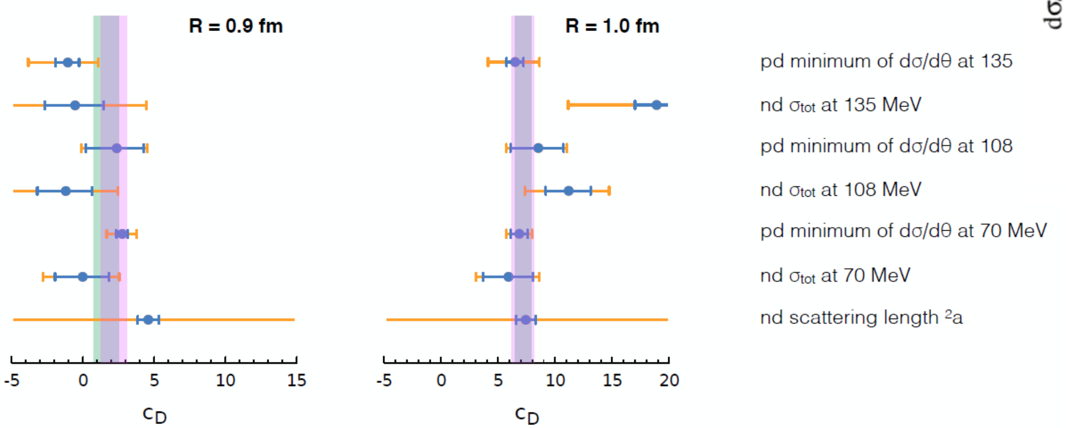


N2LO NN + N2LO 3NF



truncation error



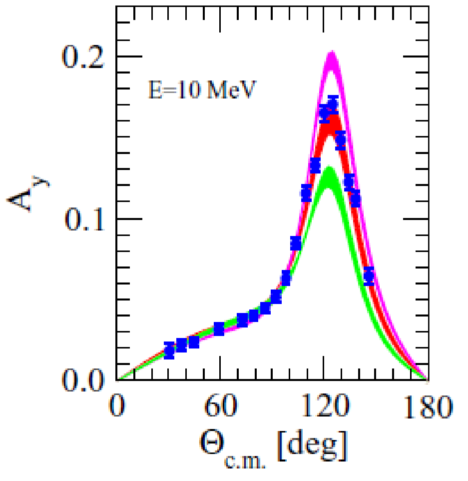
Exp:
K. Sekiguchi et al., Phys. Rev. C 65, 034003 (2002).

A_y : chiral nonlocal regularization vs chiral SCS

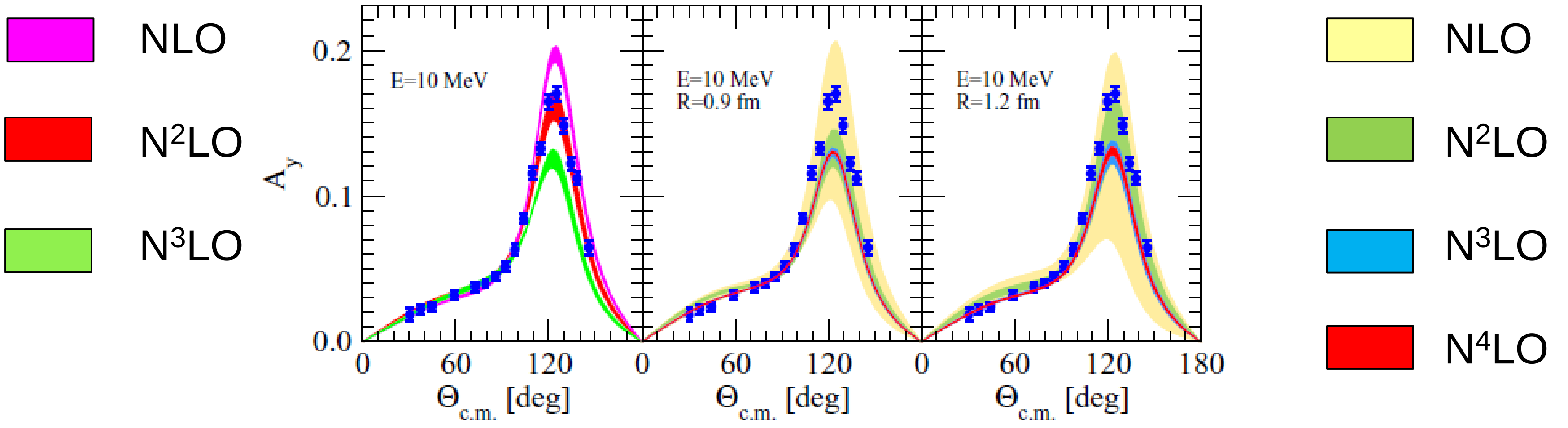
NLO

N^2LO

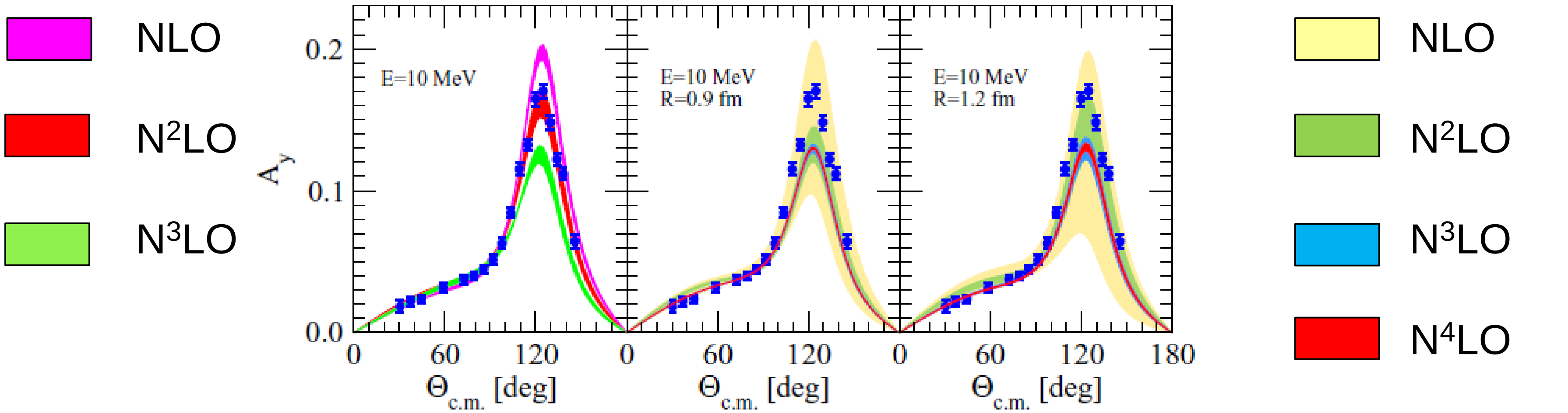
N^3LO

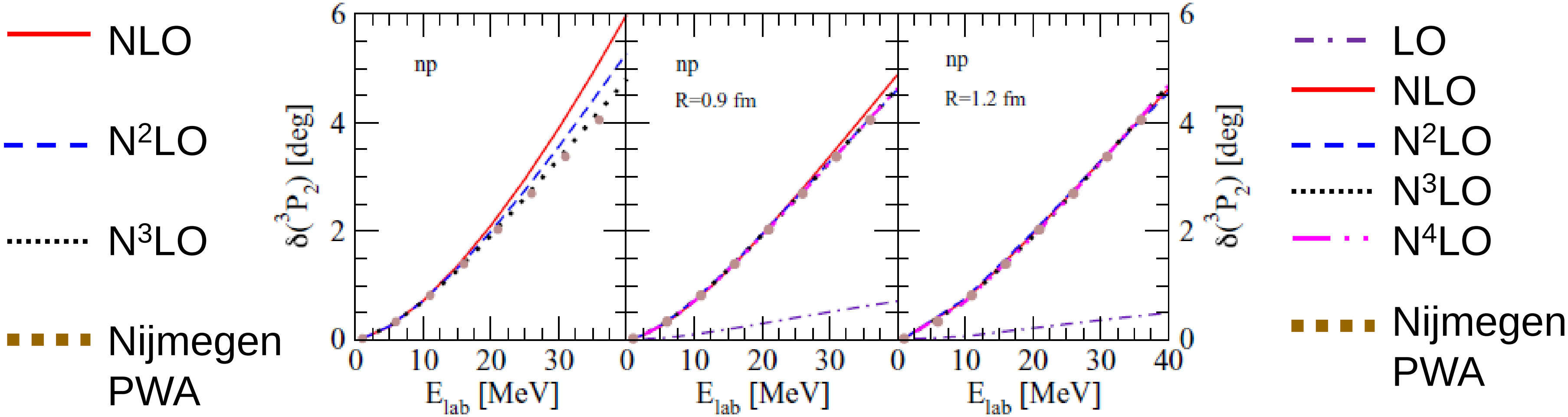


A_y : chiral nonlocal regularization vs chiral SCS

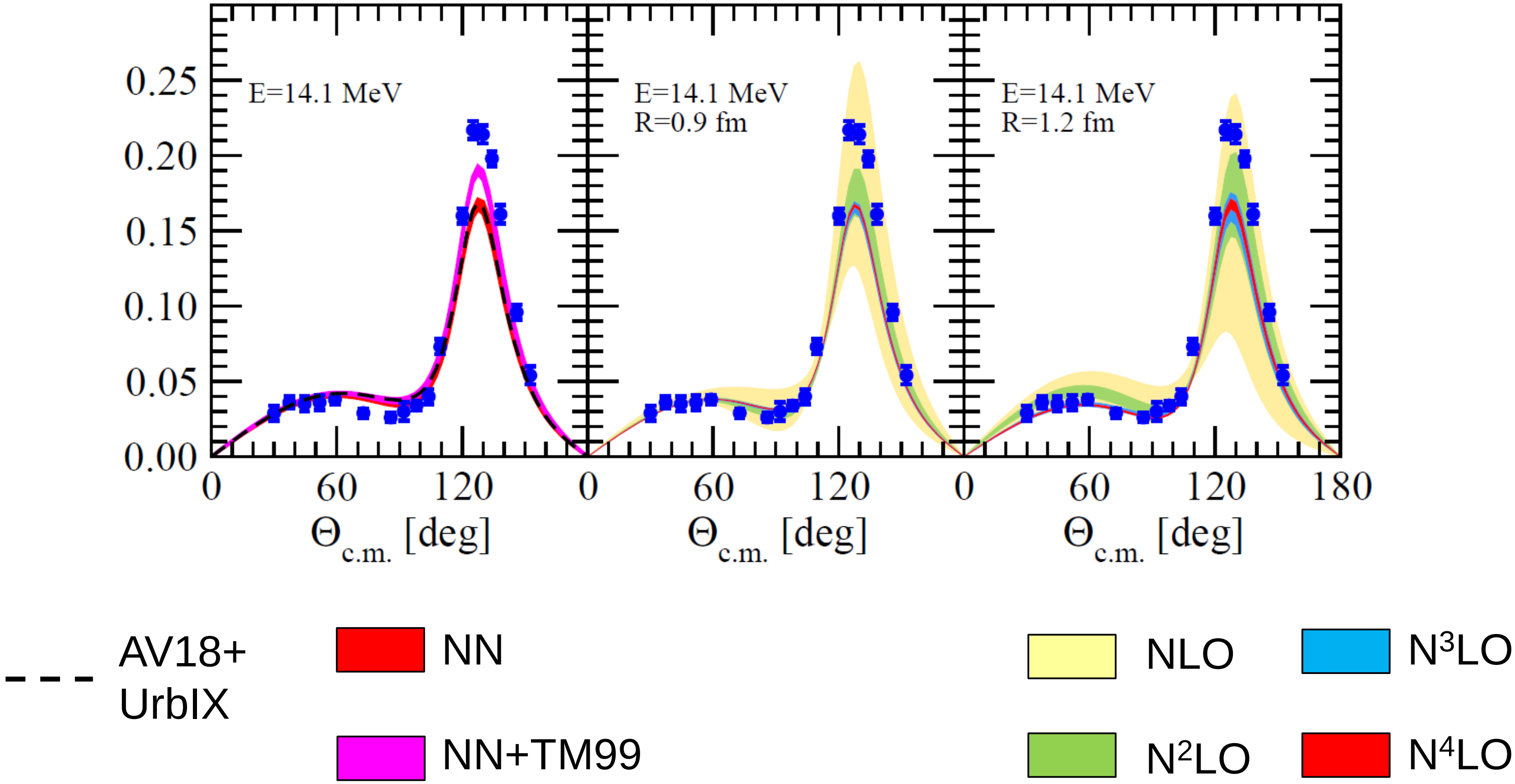


A_y : chiral nonlocal regularization vs chiral SCS





A_y : phenomenology vs chiral SCS NN



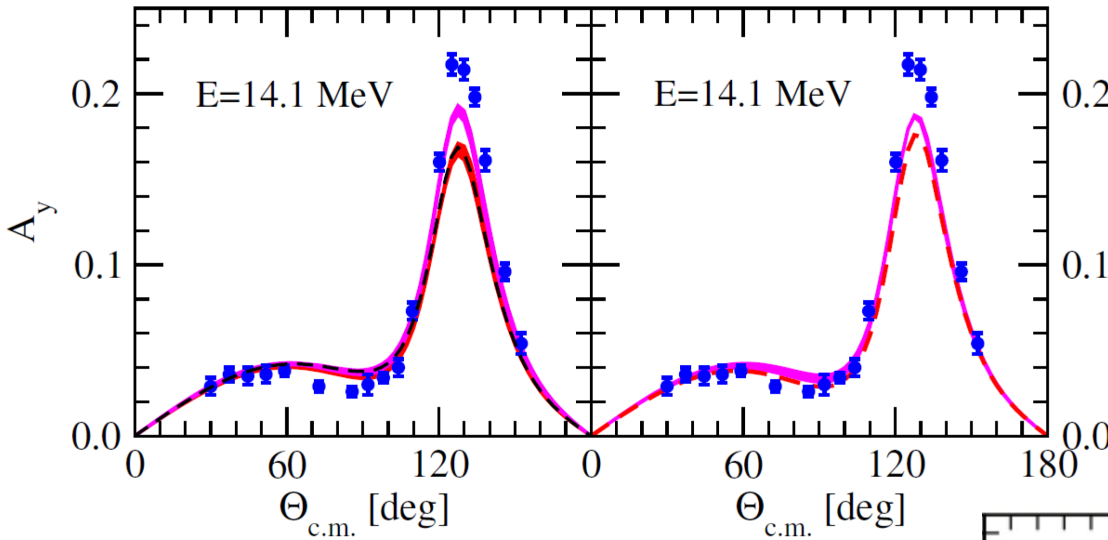
→ Chiral SCS force behaves in similar way to semi-phenomenological forces.

A_y : phenomenology vs chiral SCS NN+3NF

NN

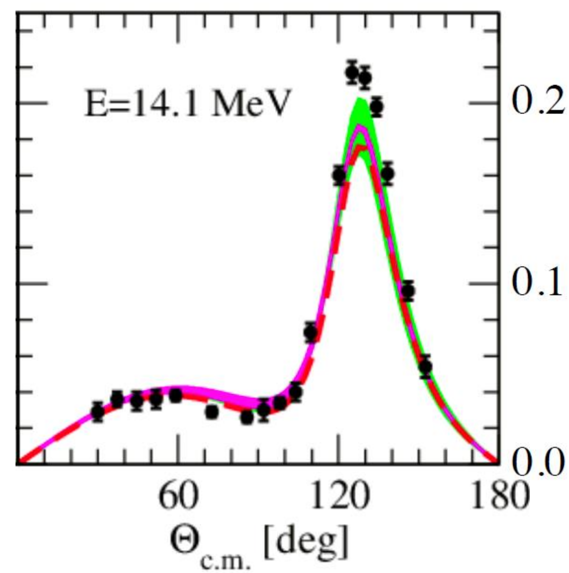
NN+ TM99

AV18+ UrbIX



N^2LO SCS NN

N^2LO SCS NN+3NF

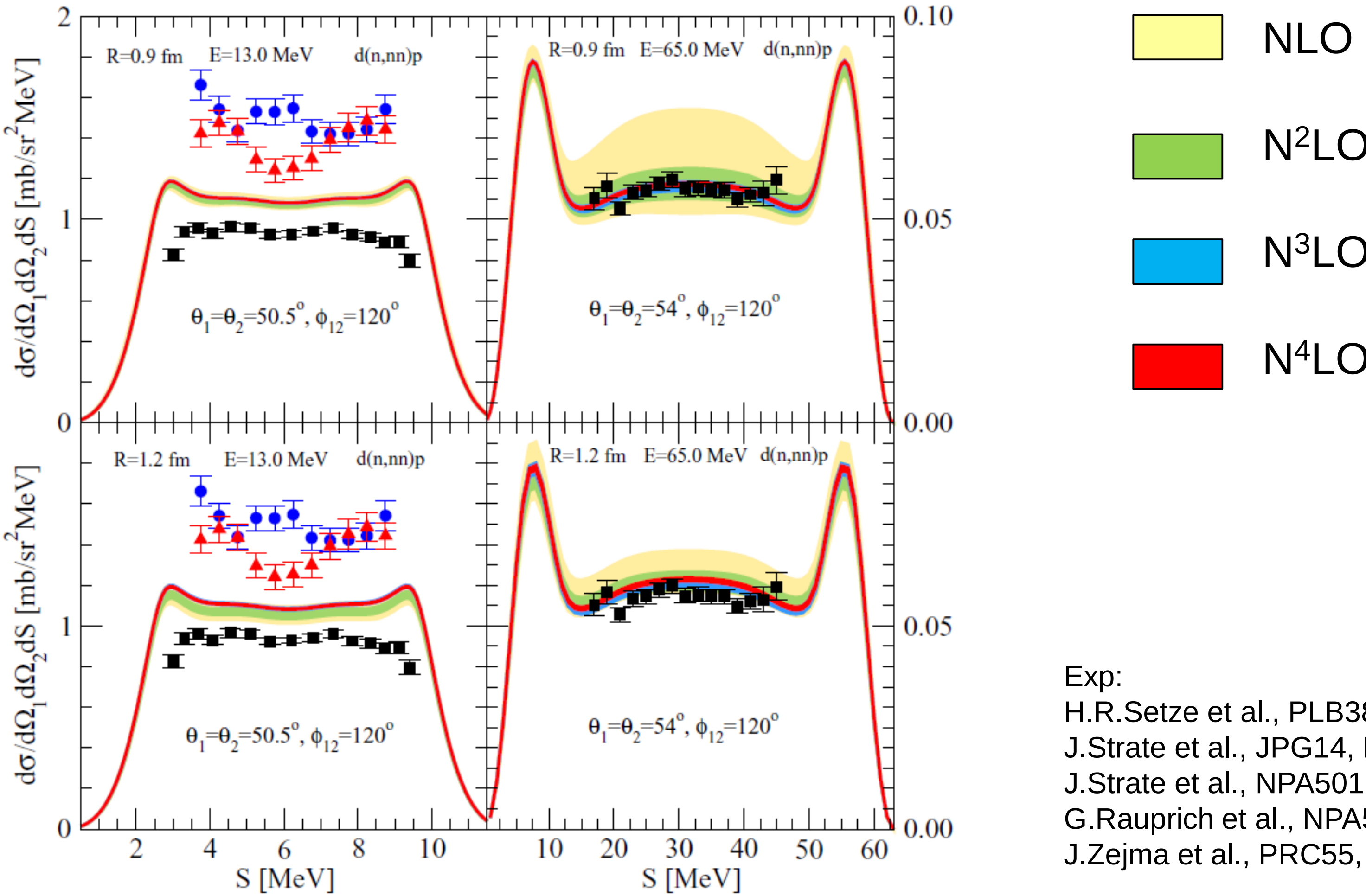


In the right panel R=0.9 fm
c_D=-2.0,…,6.0 were used for N^2LO SCS NN+3NF

truncation error for c_D=2.0

Exp: W.Tornow et al., PRC27, 2439 (1983)

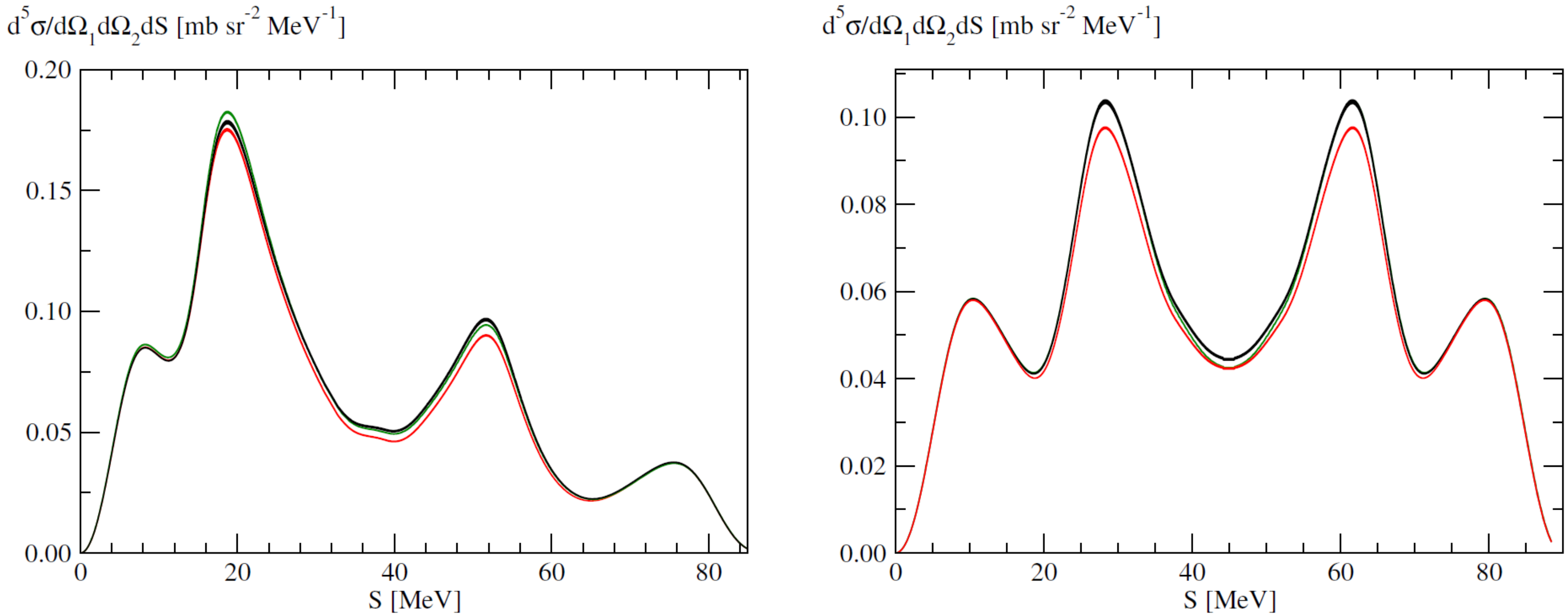
SST configuration at E=13 MeV and 65 MeV with chiral SCS, R=0.9 fm and R=1.2 fm



Exp:
H.R.Setze et al., PLB388, 229 (1996)
J.Strate et al., JPG14, L229 (1988)
J.Strate et al., NPA501, 51 (1989)
G.Rauprich et al., NPA535, 313 (1991)
J.Zejma et al., PRC55, 42 (1997)

Deuteron breakup at E=65 MeV - statistical uncertainties

Θ_1=45.0^o φ_1=0^o Θ_2=75.6^o φ_2=180^o Θ_1=59.5^o φ_1=0^o Θ_2=59.5^o φ_2=180^o

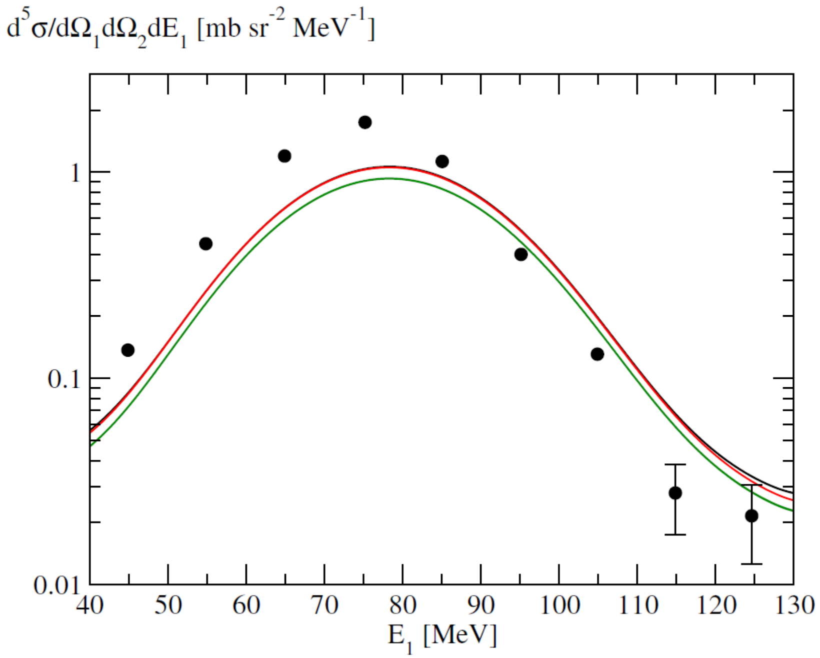


N^2LO SMS N^4LO SMS OPE-G

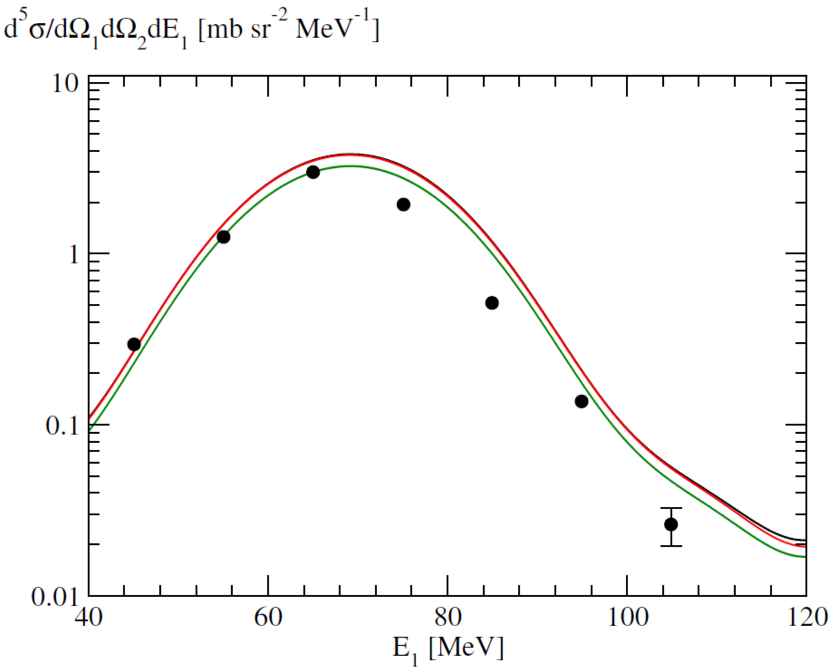
→ Statistical uncertainties are very small at E=65 MeV

Deuteron breakup at E=200 MeV - statistical uncertainties

Θ_1=45.0° φ_1=0° Θ_2=35.0° φ_2=180°



Θ_1=52.0° φ_1=0° Θ_2=35.0° φ_2=180°



N^2LO SMS N^4LO SMS   OPE-G

→ Statistical uncertainties remain small also at E=200 MeV

Exp: W. Pairsuwan, et al., Phys. Rev. C52, 2552 (1995).

Summary

- New models of the chiral nuclear forces derived recently have been applied to the nucleon-deuteron scattering up to E=200 MeV.
- We find a **good data description**, **however** at the orders of chiral expansion investigated at the moment (with NN force up to N^4LO and NN+3NF up to N^2LO) **none of three-nucleon puzzles is solved.**

- New semilocal regularizations, both in coordinate as well as in momentum spaces, lead to **significantly smaller cut-off dependence** then older generation of Bochum-Bonn potentials. Especially, for the SMS force this dependence is so weak that **the problem of cut-off dependence (from 3N scattering application perspective) is solved.**
- **Statistical errors** can be also estimated for the SMS chiral interaction. We conclude that resulting uncertainty is **smaller than** truncation errors.
- Consistent SCS NN and 3N N^2LO potentials give predictions of the similar quality as semi-phenomenological interactions.