^{3}H and ^{3}He bound state calculations without angular momentum decomposition

Kacper Topolnicki

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1 XI 2018

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1 TRADITIONAL APPROACH

2 3D APPROACH

3 CURRENT STATE OF 3D CALCULATIONS

4 RESULTS FOR 3N BOUND STATE

5 SUMMARY AND OUTLOOK

PW CALCULATIONS - IN A NUTSHELL

- Work in a finite sized basis of - angular momentum, spin, isospin eigenstates
- QM operators can be represented using matrices with finite size . . .

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PW CALCULATIONS - IN A NUTSHELL

- $\check{X}$ some 2N operator ($\check{}$ denotes operators) eg.: 2N potential ($\check{V}$), 2N transition operator (for given energy $\check{t}(E)$), free propagator (for a given energy $G_0(E)$), ...
- Each ■ in subspace with given orbital angular momentum l, spin s and total angular momentum j
- Different momentum magnitude states, and different projections of total angular momentum $\langle |\boldsymbol{p}'|(l's')j'm_{j'} \mid \ldots \mid |\boldsymbol{p}|(ls)jm_j \rangle$
- Impose pairity, time reversal and rotational symmetry ...

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- Different coupling schemes for three, four, . . . particles
- Implementation requires heavily oscilating functions

$$Y_l^m(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

- Calculations need to include many partial waves in order to acheve convergence
  - High energies
  - Long range interactions (Coulomb!)

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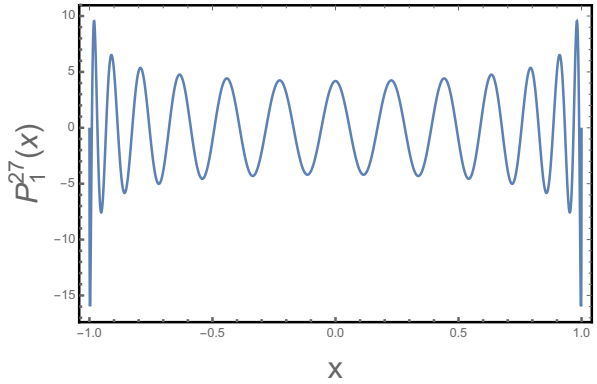

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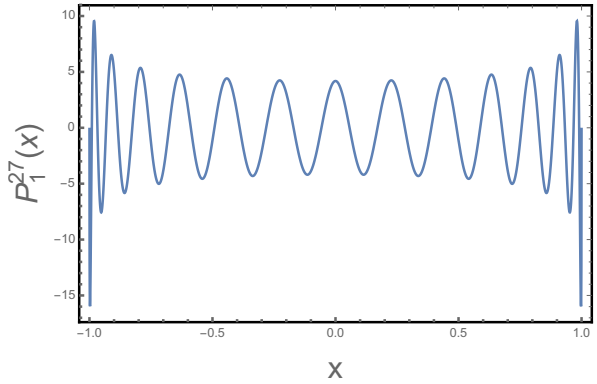

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3D CALCULATIONS - IN A NUTSHELL

- Example: the 2N transition operator $\check{t}$.
- Assume we are working in momentum space with $\boldsymbol{p}' = (p'_x, p'_y, p'_z)$ being the final and $\boldsymbol{p} = (p_x, p_y, p_z)$ being the initial momentum of the two nucleons.
- Solve the LSE: $\check{t} = \check{V} + \check{V}\check{G}_0\check{t}$

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- Ignoring isospin:

$$\left[\langle \boldsymbol{p}' \mid \check{t} \mid \boldsymbol{p} \rangle\right] =$$

$$\begin{bmatrix} t_{11}(p'_x, p'_y, p'_z, p_x, p_y, p_z) & t_{12}(\ldots) & t_{13}(\ldots) & t_{14}(\ldots) \\ t_{21}(\ldots) & t_{22}(\ldots) & t_{23}(\ldots) & t_{24}(\ldots) \\ t_{31}(\ldots) & t_{32}(\ldots) & t_{33}(\ldots) & t_{34}(\ldots) \\ t_{41}(\ldots) & t_{42}(\ldots) & t_{43}(\ldots) & t_{44}(\ldots) \end{bmatrix}$$

- $\check{V}$, $\check{G}_0$ in a similar form ($\check{G}_0$ - diagonal)
- Very wastefull - we need to calculate 16 functions $t_{ij}(p'_x, p'_y, p'_z, p_x, p_y, p_z)$ of 6 real parameters that satisfy the LSE

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- The matrix element in momentum space that satisfies appropriate symmetries (parity inversion, time reversal, particle exchange) can be written [Phys. Rev. 96 1654 (1954)] as a linear combination of 6 scalar functions t_i and spin operators $[w_i]$:

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^{3}H, ^{3}He - FADDEEV EQUATION

- Ingredients: bound state energy E ; free propagator $\check{G}_0(E)$; $\check{V}$ two nucleon potential between particles $2, 3$; $\check{V}^{(1)}$ three nucleon potential symmetric with respect to the exchange of particles $2, 3$; particle exchange operator P_{ij}.
- Faddeev component of the 3N bound state:

$$| \psi\rangle = \check{G}_0(E) \left(\check{V} + \check{V}^{(1)}\right) \left(1 + \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}\right) | \psi\rangle$$

- Full bound state wave function:

$$| \Psi\rangle = \left(1 + \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}\right) | \psi\rangle$$

OPERATOR FORM OF 3N STATE

- Ingredients: Jacobi momenta $\boldsymbol{p}$, $\boldsymbol{q}$; two particle subsystem isospin t ; total isospin T ; total isospin projection M_T ; given operators in the spin space of the 3N system $\check{O}_i$; spin state $\chi_m = \mid (1\frac{1}{2})\frac{1}{2}m\rangle$
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$$\langle \boldsymbol{pq}; (t\frac{1}{2})TM_T \mid \psi\rangle = \sum_{tT}\sum_{i=1}^{8} \psi_{tT}^{(i)}(p, q, \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})\, \check{O}_i(\boldsymbol{p}, \boldsymbol{q}) \mid \chi_m\rangle$$

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- Ingredients: scalar functions that define the 3N state $\phi \in span(\{\phi_i^{t^T}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})\})$; energy dependent operator $\check{A}(E)$
- The transformation:

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- Actually solve $\check{A}(E)\psi = \lambda\psi$, try various energies, search for $\lambda = 1$
- Use Arnoldi algorithm
  - Solve in Krylov subspace:

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  - Initially: $\phi^{(i)}_{0,tT}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}}) = \pm\phi^{(i)}_{0,tT}(p, q, -\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})$
  - The algorithm requires the implementation of $\check{A}$ and scalar product between scalar functions
  - End result: $N \times N$ matrix representation of $\check{A}$, $N \approx 80$

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$$\check{A}(E) = \left(\check{A}_{\check{G}_0 \check{V}}(E) + \check{A}_{\check{G}_0 V^{(1)}}(E)\right) \check{A}_{\check{1}+\check{P}}$$

- For ^{3}He the 2N potential can be split into the strong and longer ranged part: $\check{V} = \check{V}_{NN} + \check{V}_C$
- Dependence on the screening radius in $\check{V}_C(R)$
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  - $\psi \stackrel{?}{=} \beta$ - all plots contain both β and ψ
- Bound state $|\Psi\rangle$:
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- Very flat dependence $\lambda(E)$
  - Calculations have to be repeated many times for different energies
- Many fold integrals
  - Reuse of already computed integrals
  - Novel method of performing the integrals
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- https://arxiv.org/abs/1810.07436v2
- Submitted to *Phys. Rev. C*
- First generation chiral NNLO 2N and 3N forces
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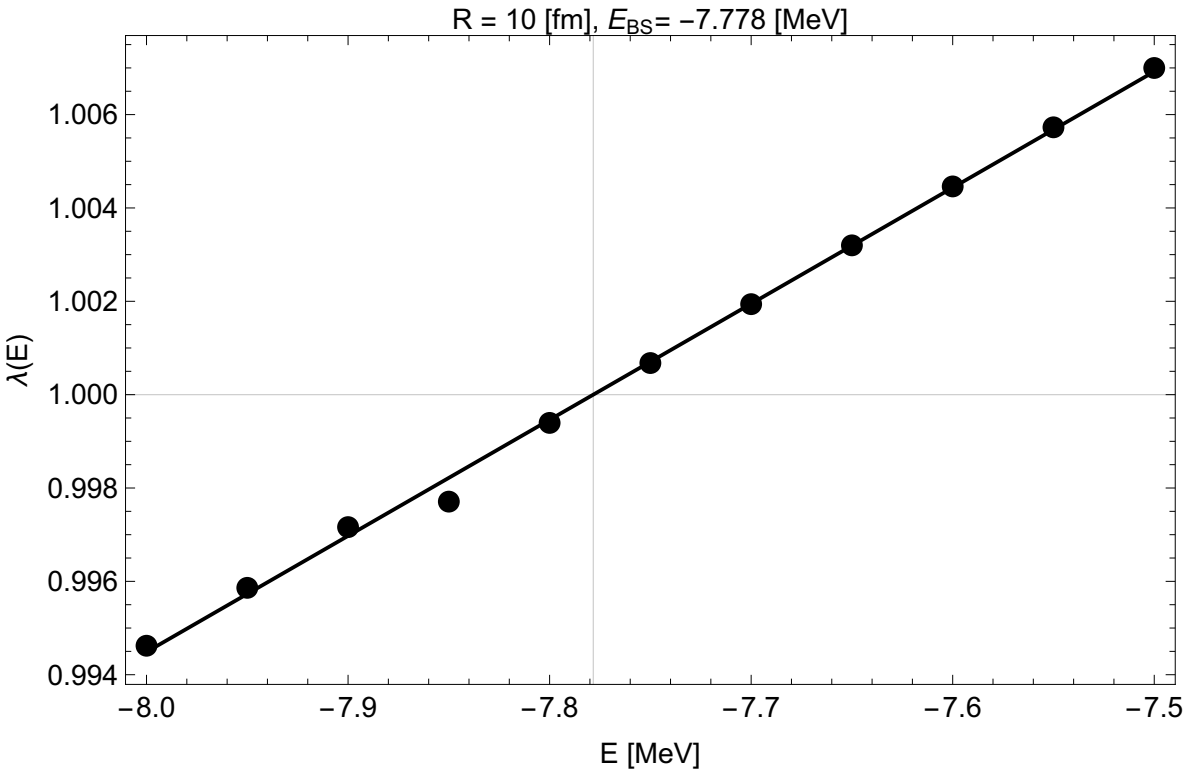

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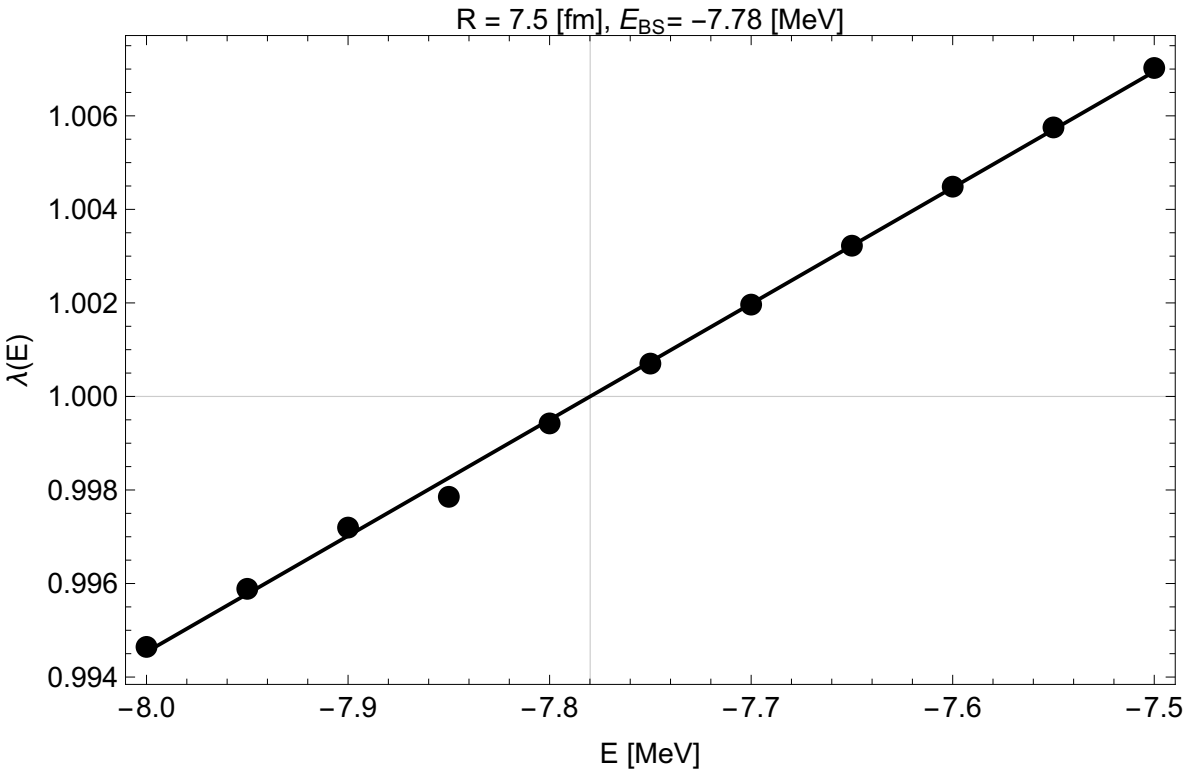

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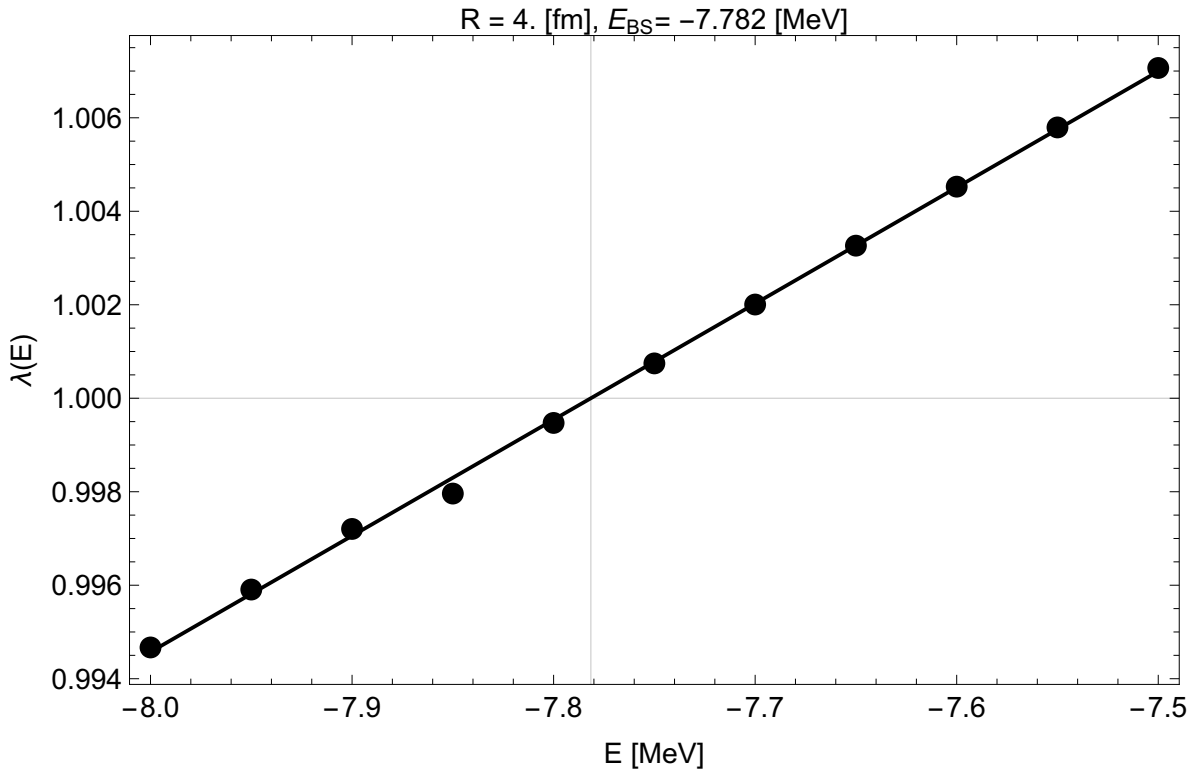

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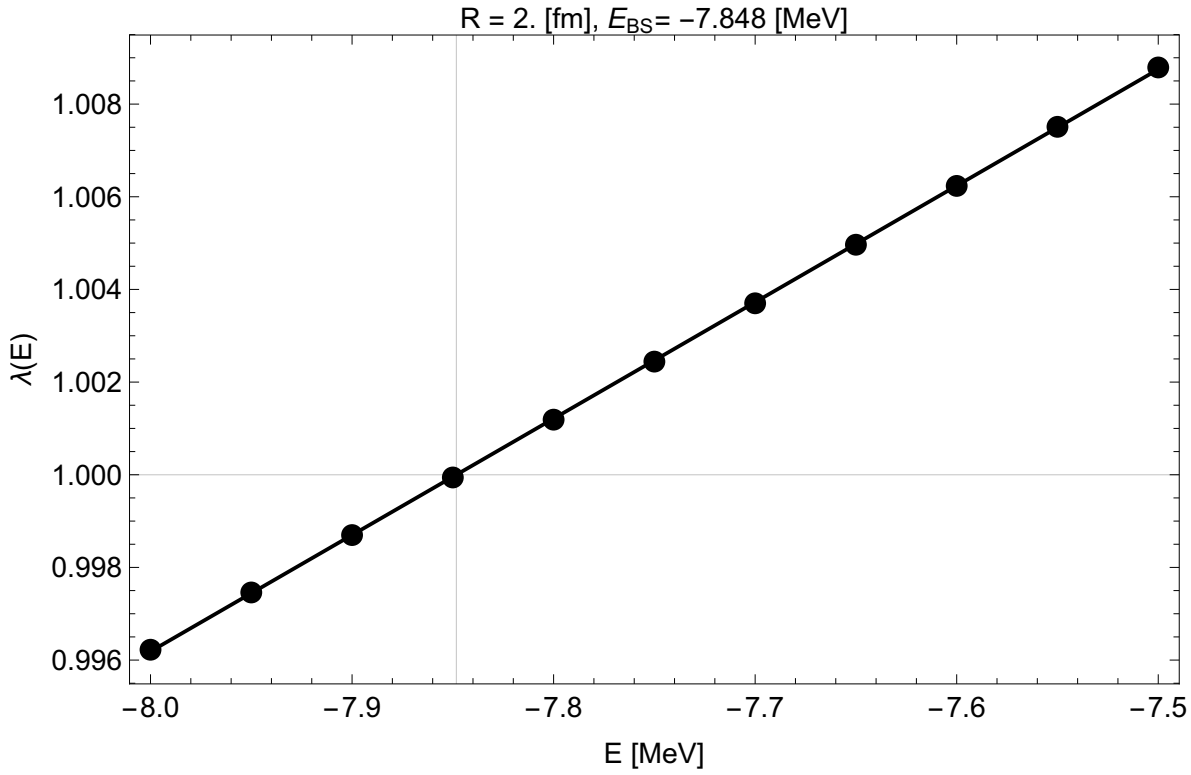

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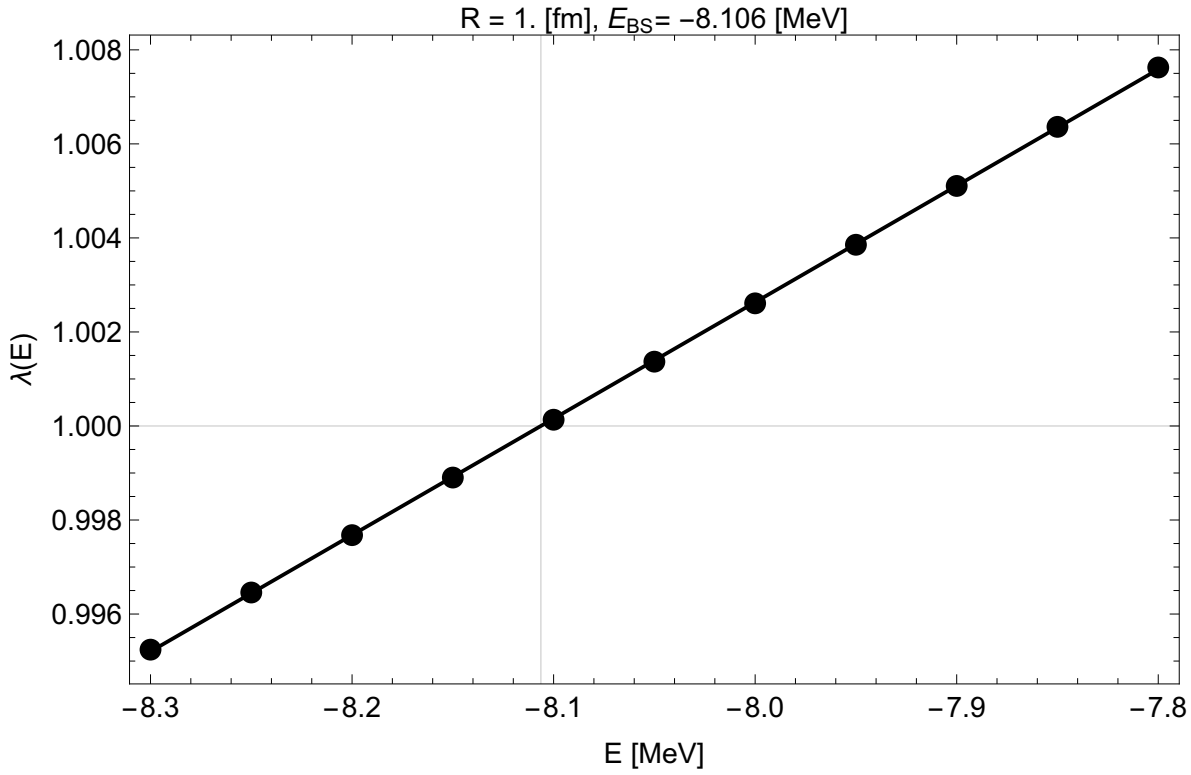

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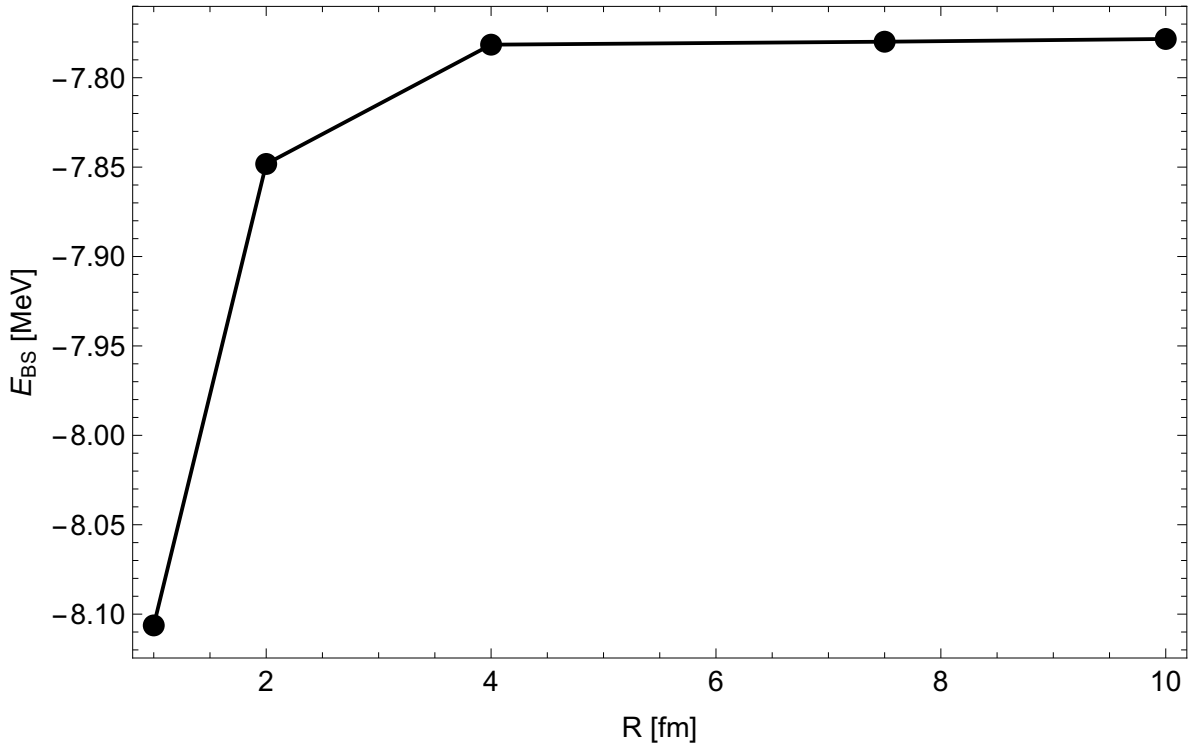

^{3}He FADDEEV COMPONENT

- Solid markers: $\psi^{(1)}(p = 0.15[\text{fm}^{-1}], q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}} = -0.88)$ as a function of q
- Empty markers: $\beta^{(1)}(p = 0.15[\text{fm}^{-1}], q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}} = -0.88)$ as a function of q
- Correct solution implies $\psi = \beta$
- Squares, circles, diamonds correspond to $t = 0, T = \frac{1}{2}$; $t = 1, T = \frac{1}{2}$; $t = 1, T = \frac{3}{2}$
- Screening radius: 10[fm]
- Bound state energy: -7.72654[MeV]

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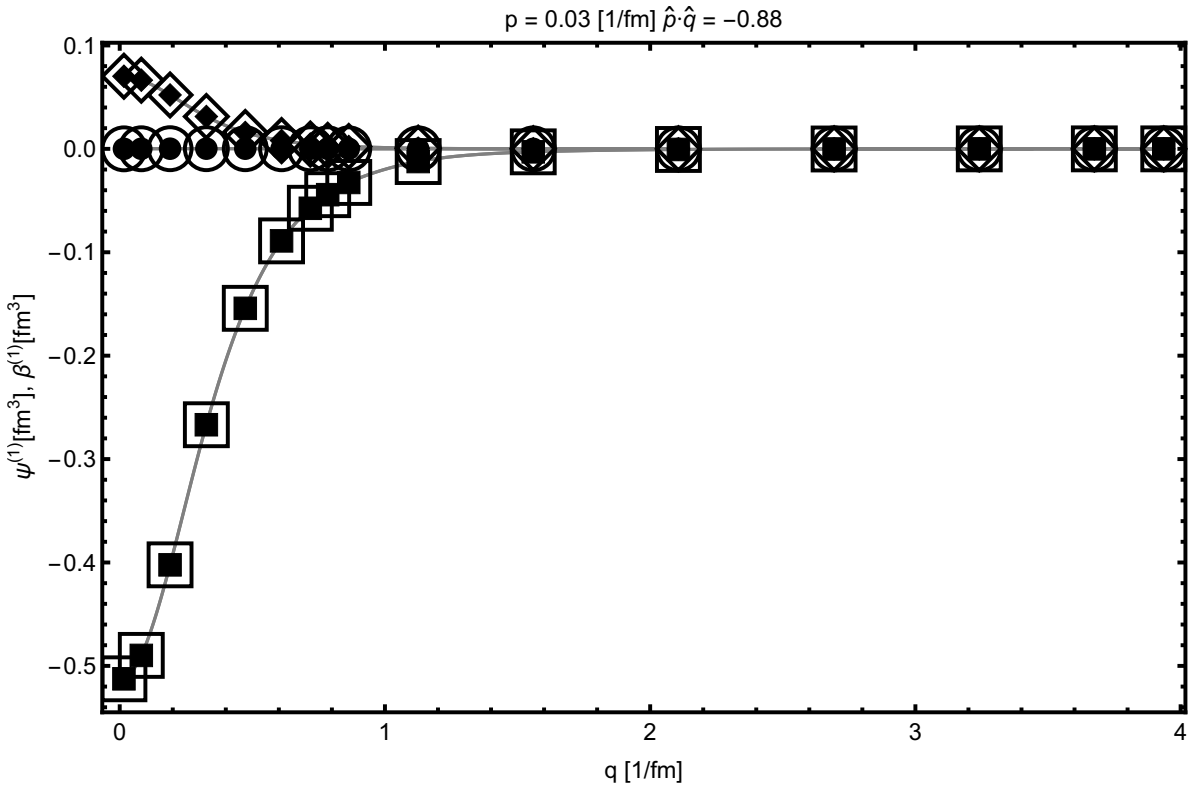

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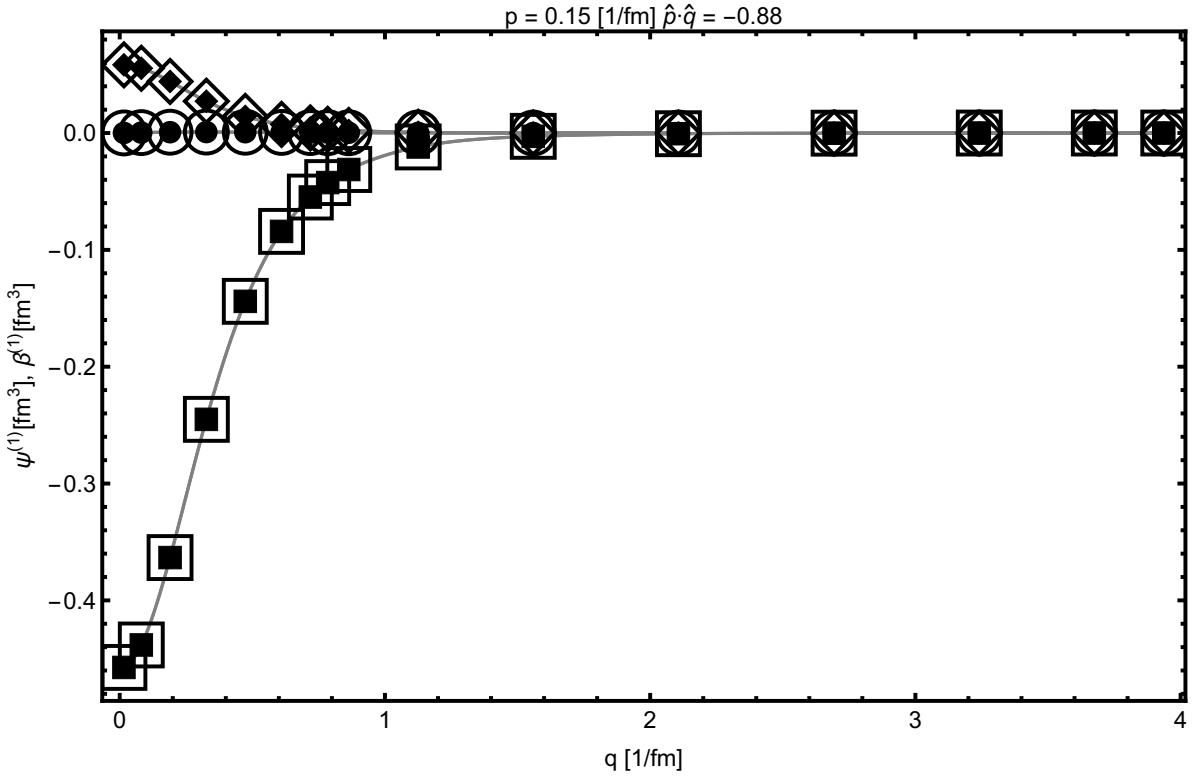

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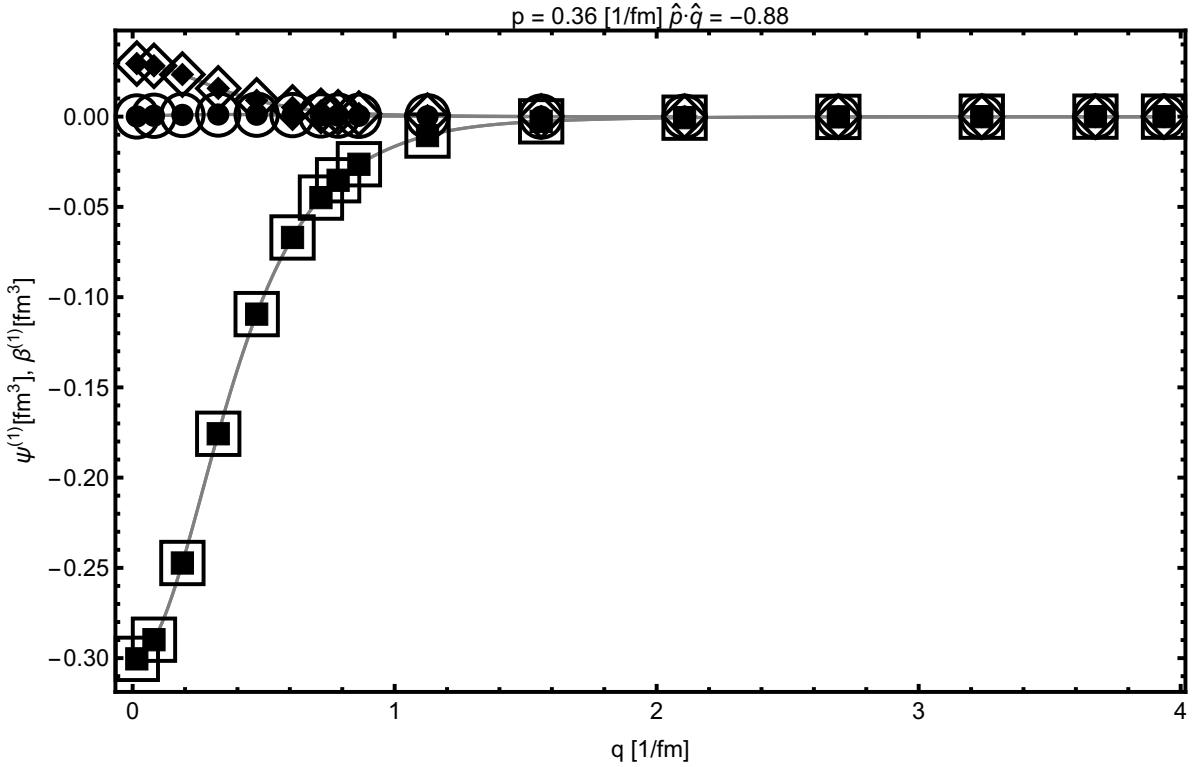

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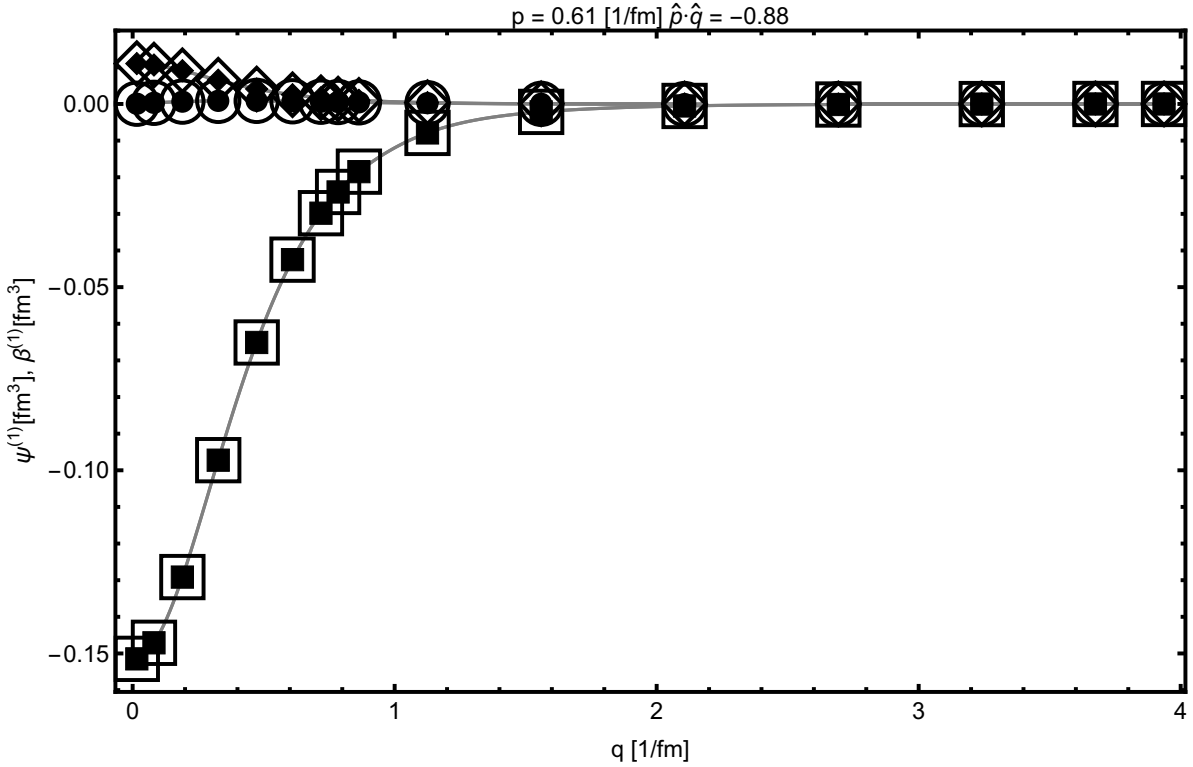

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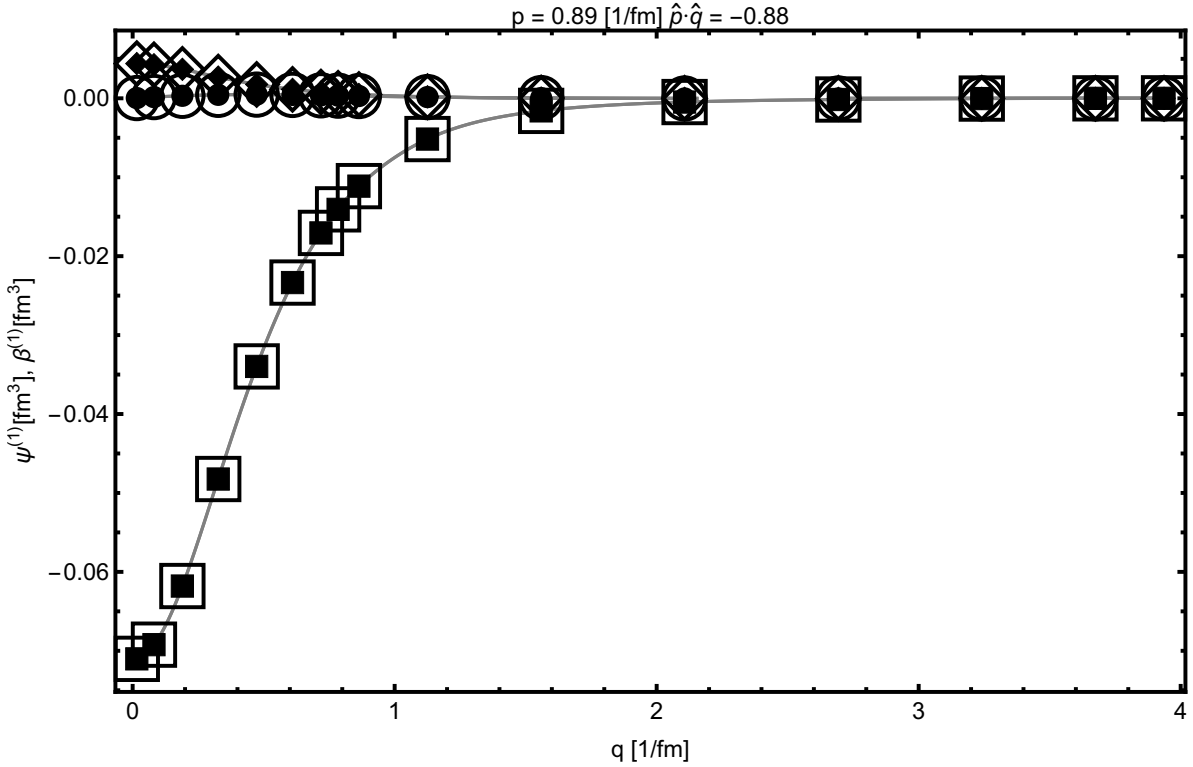

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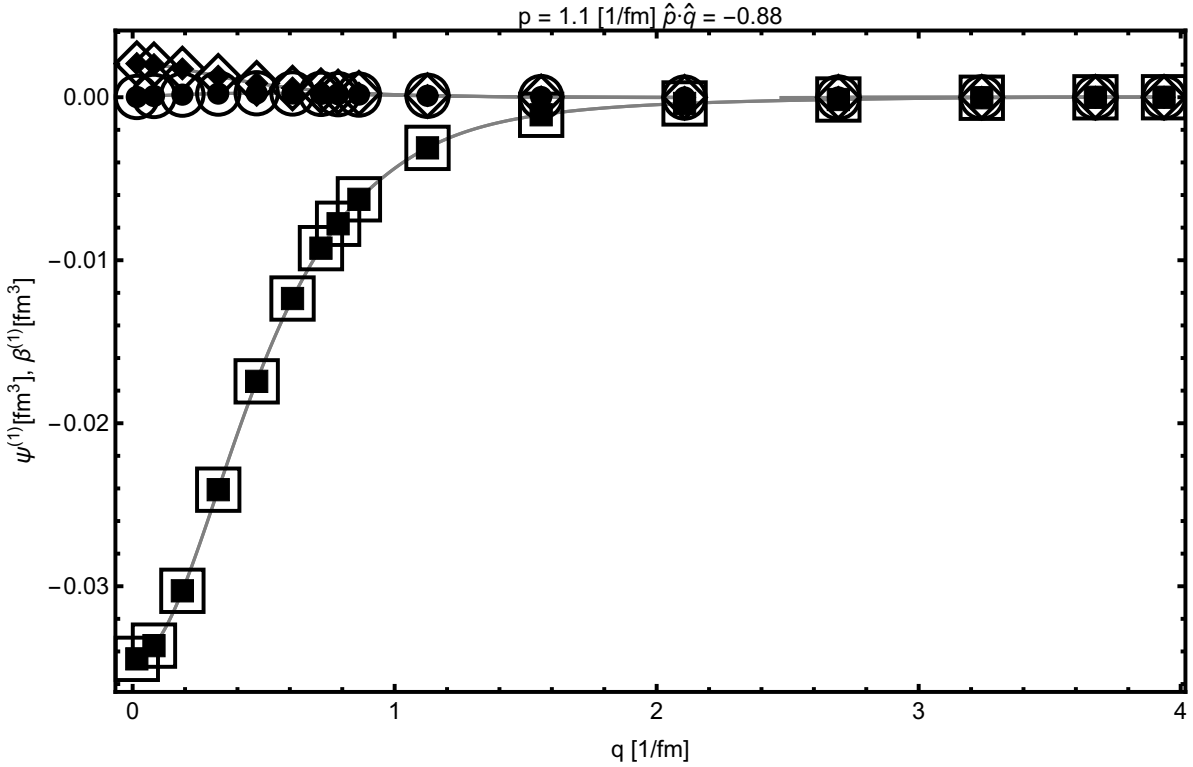

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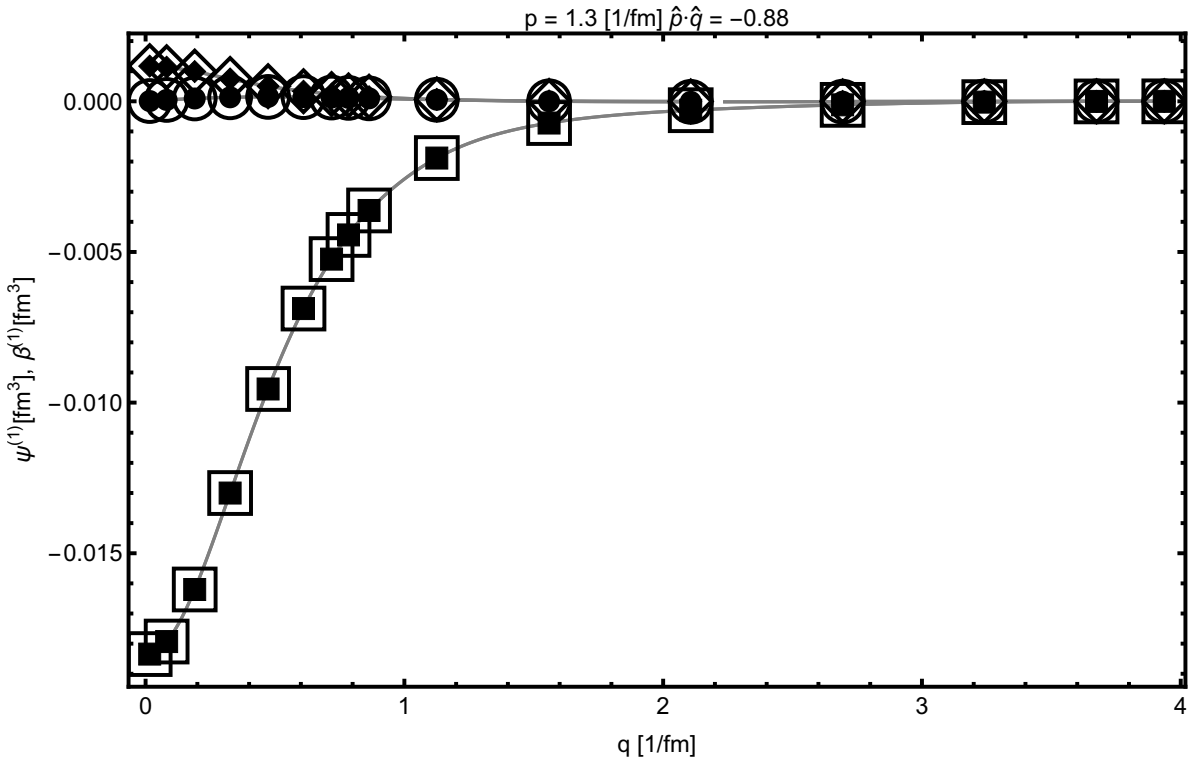

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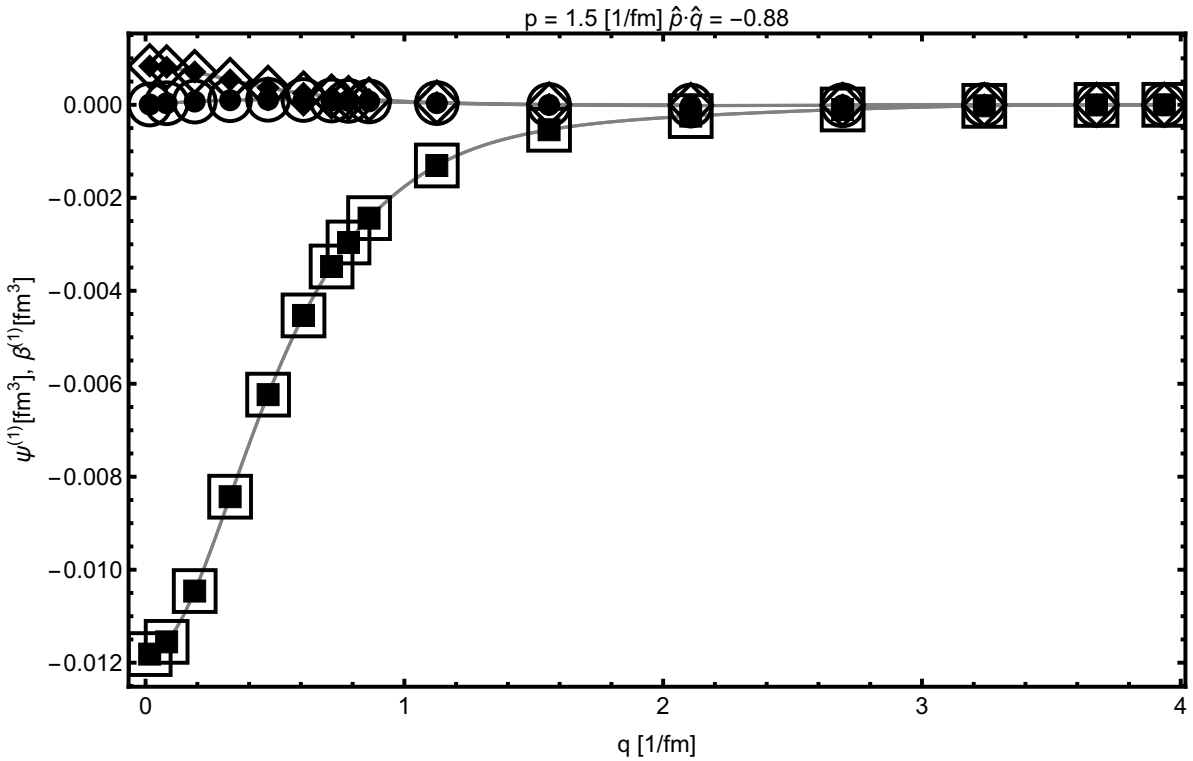

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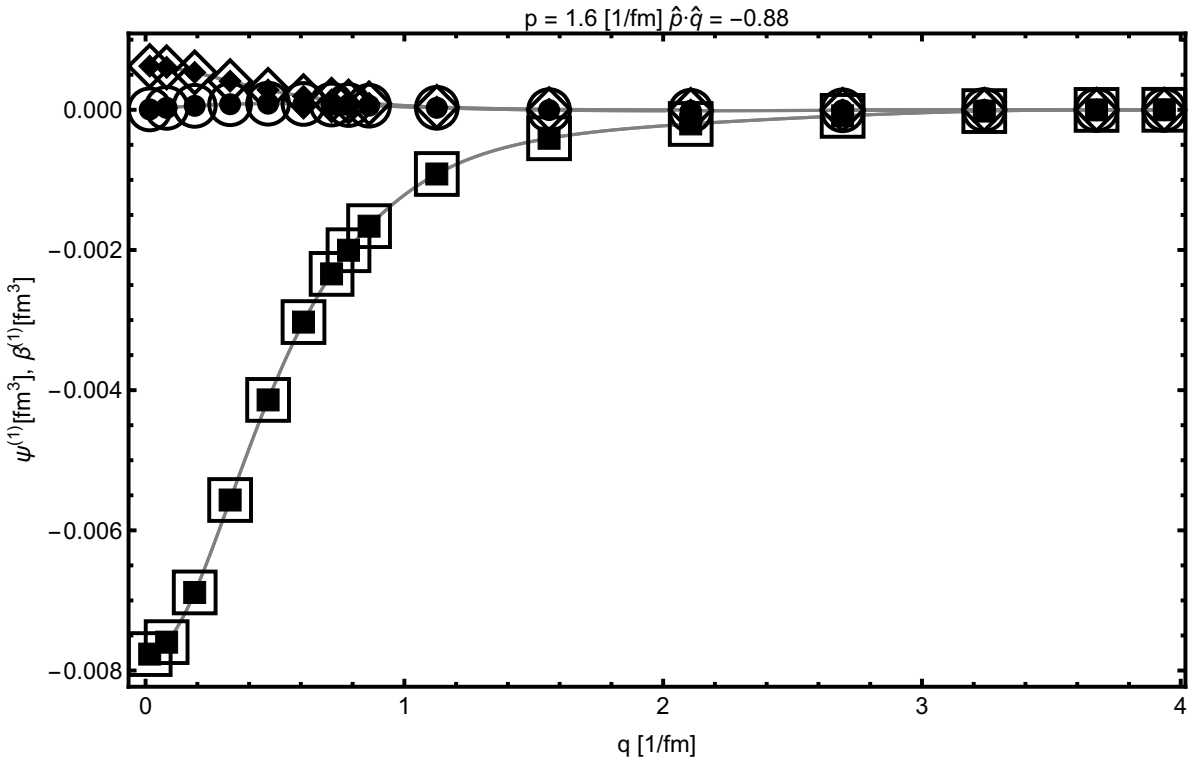

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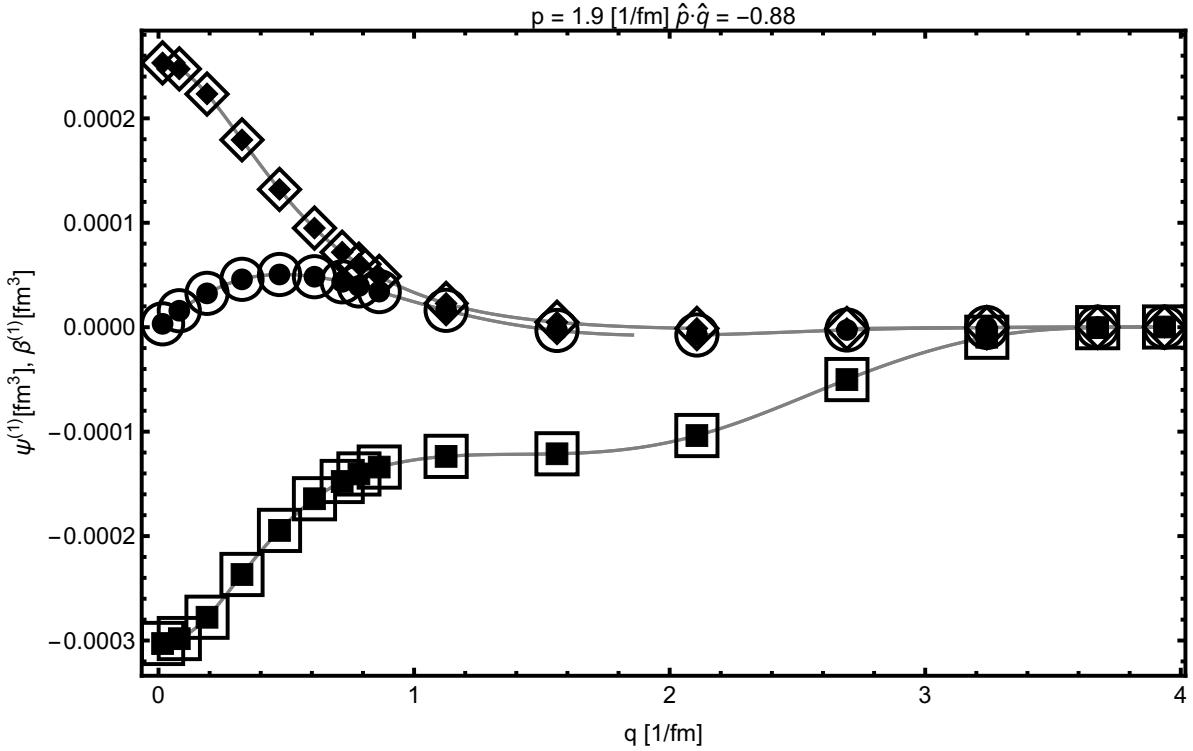

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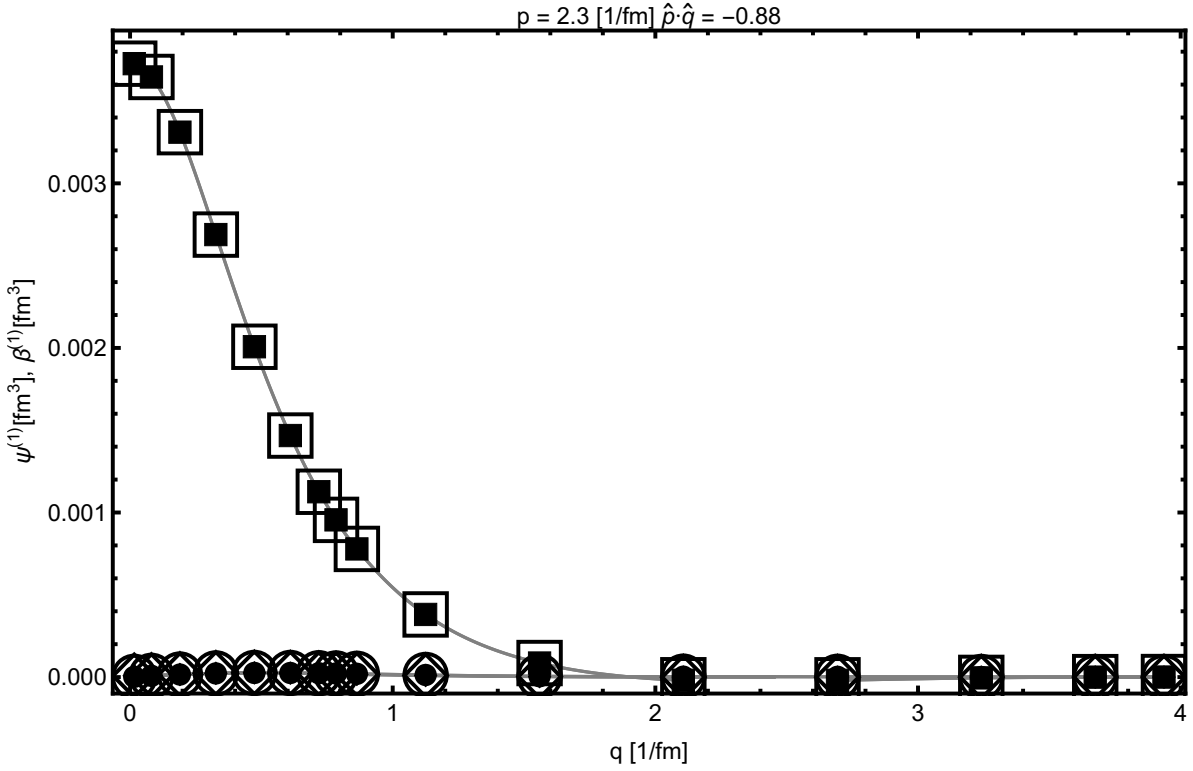

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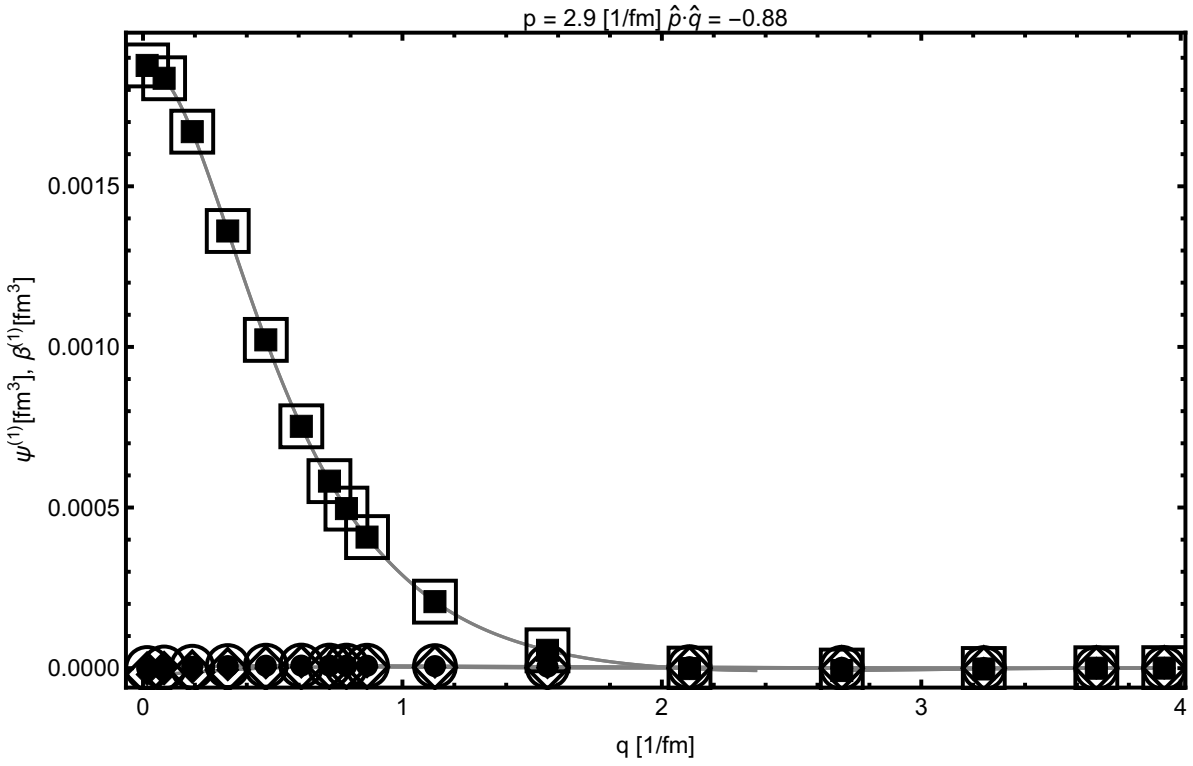

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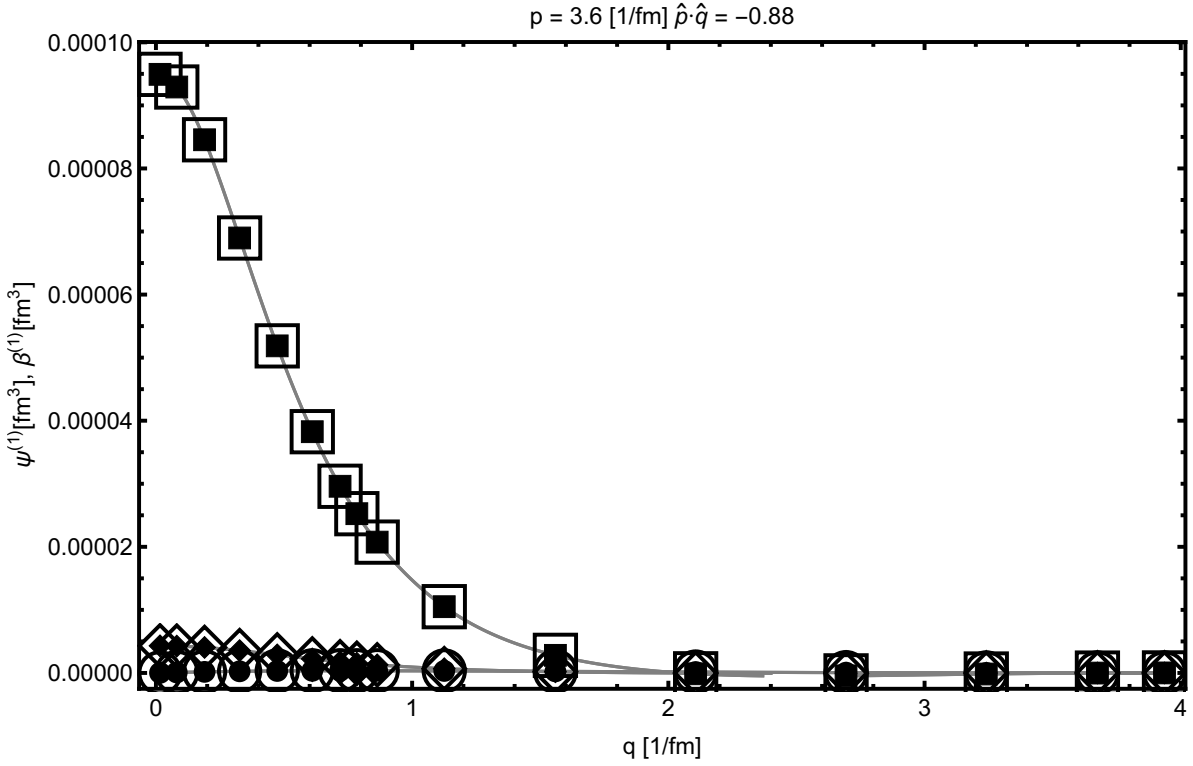

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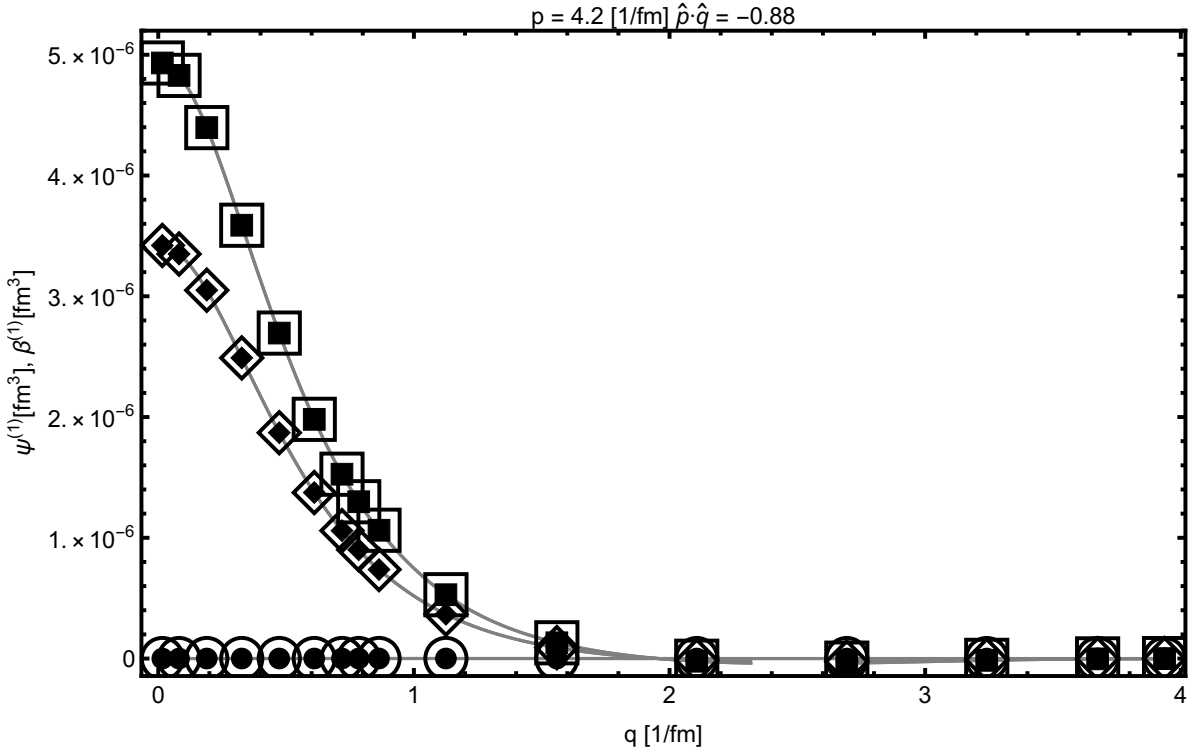

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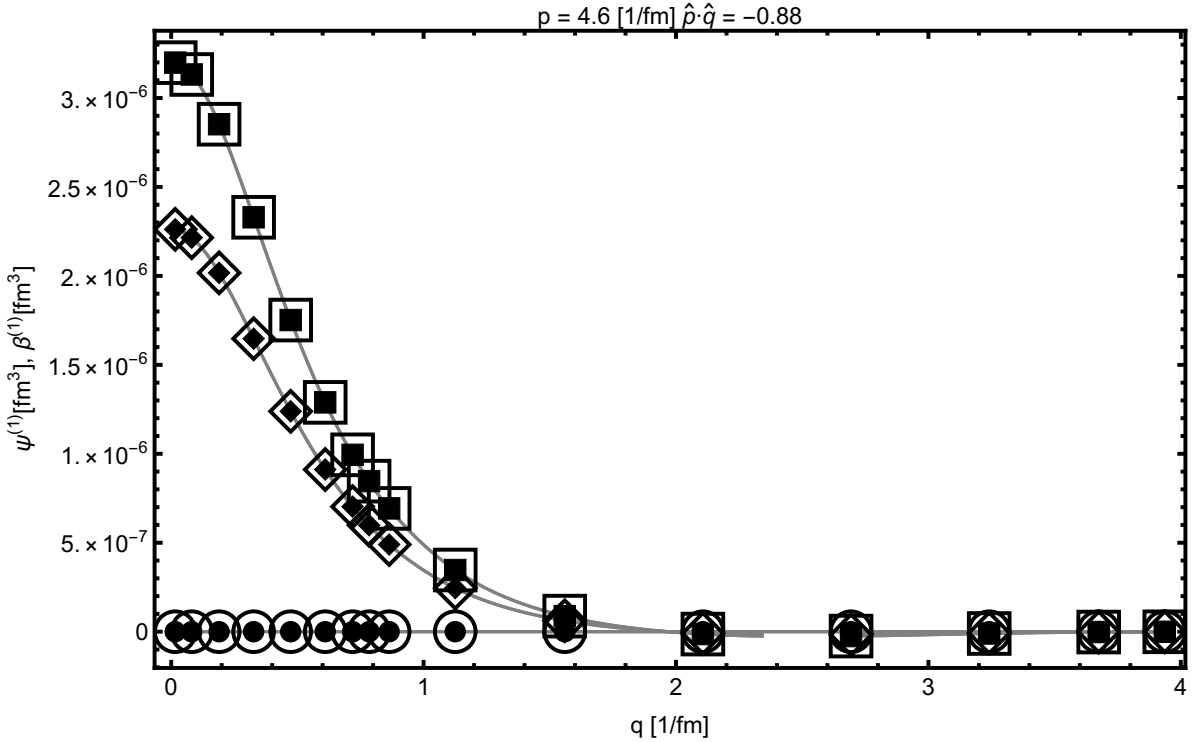

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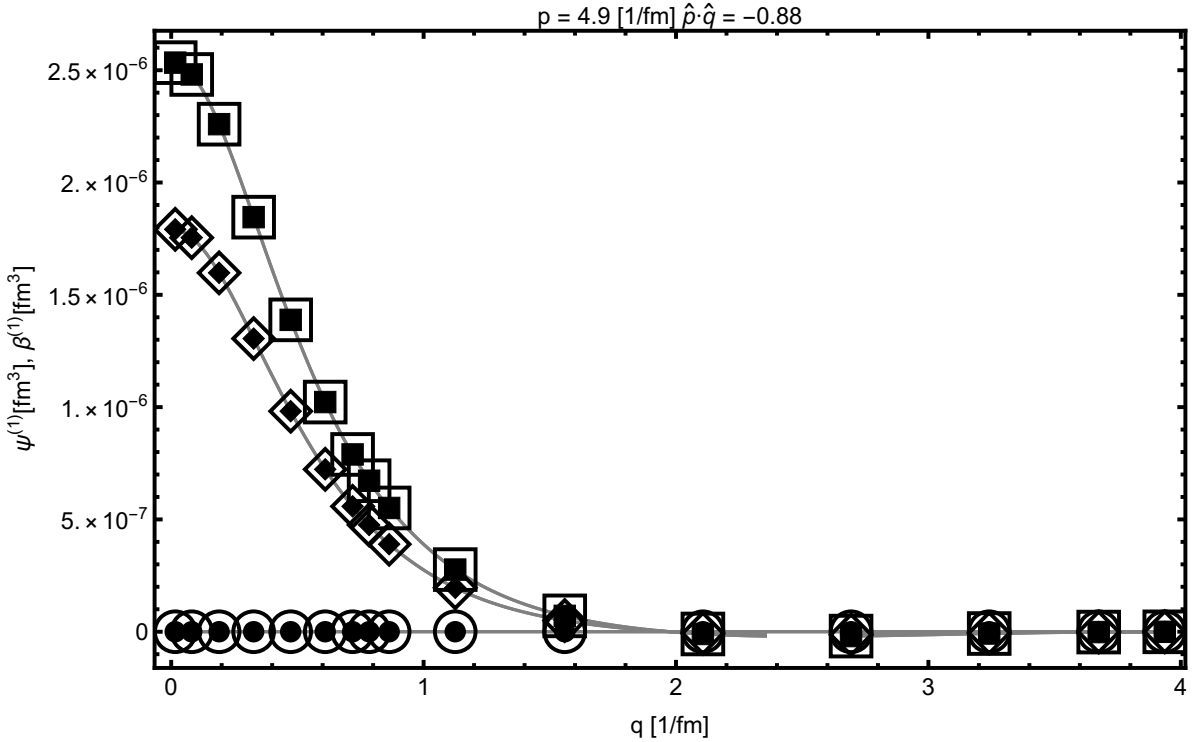

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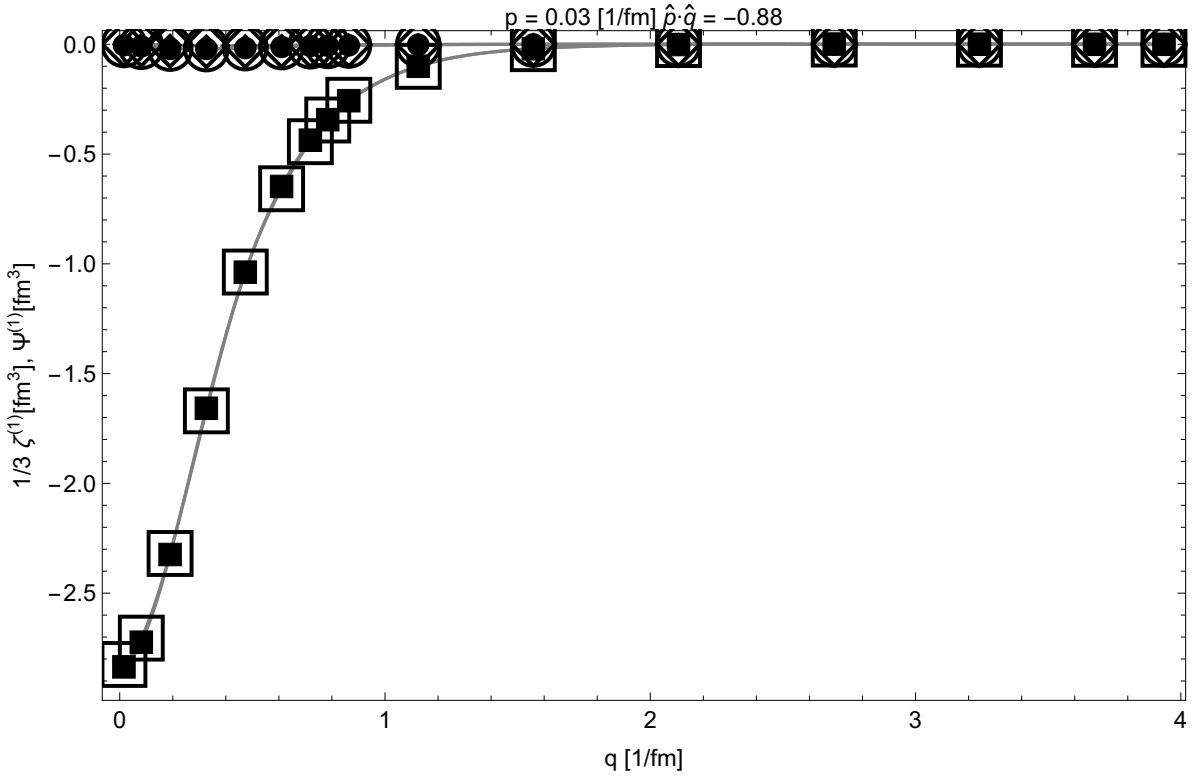

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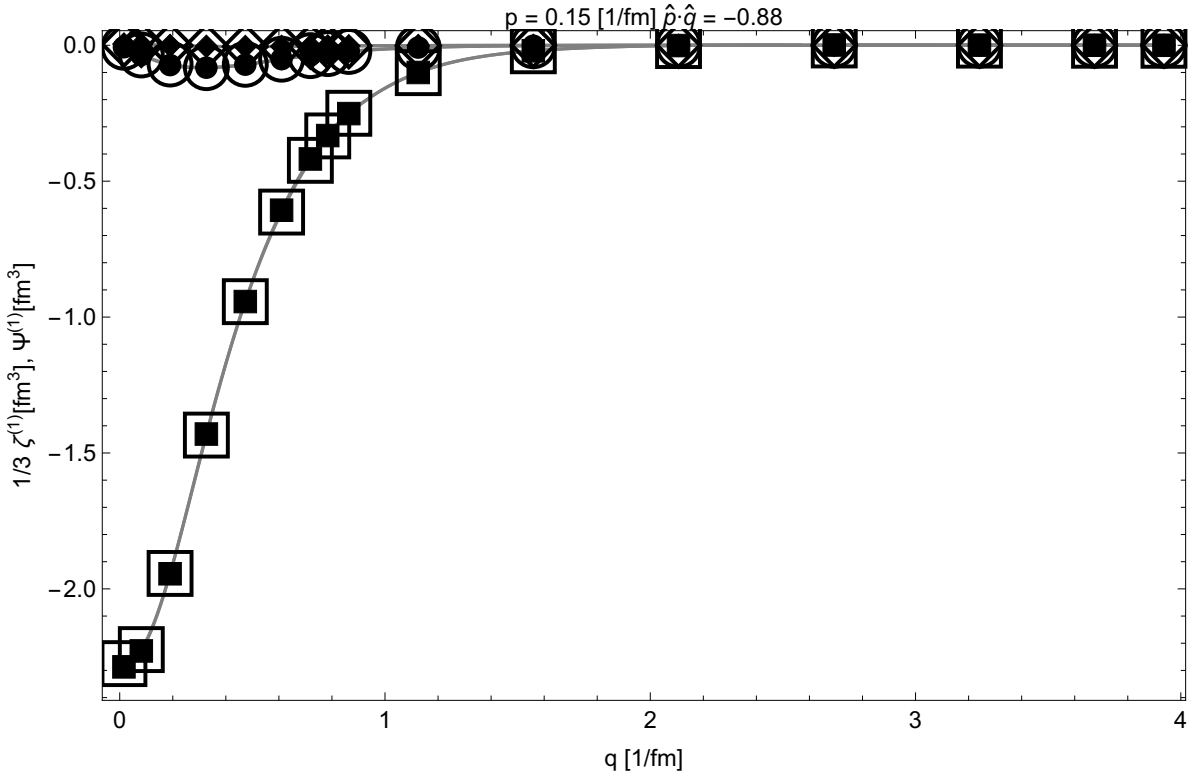

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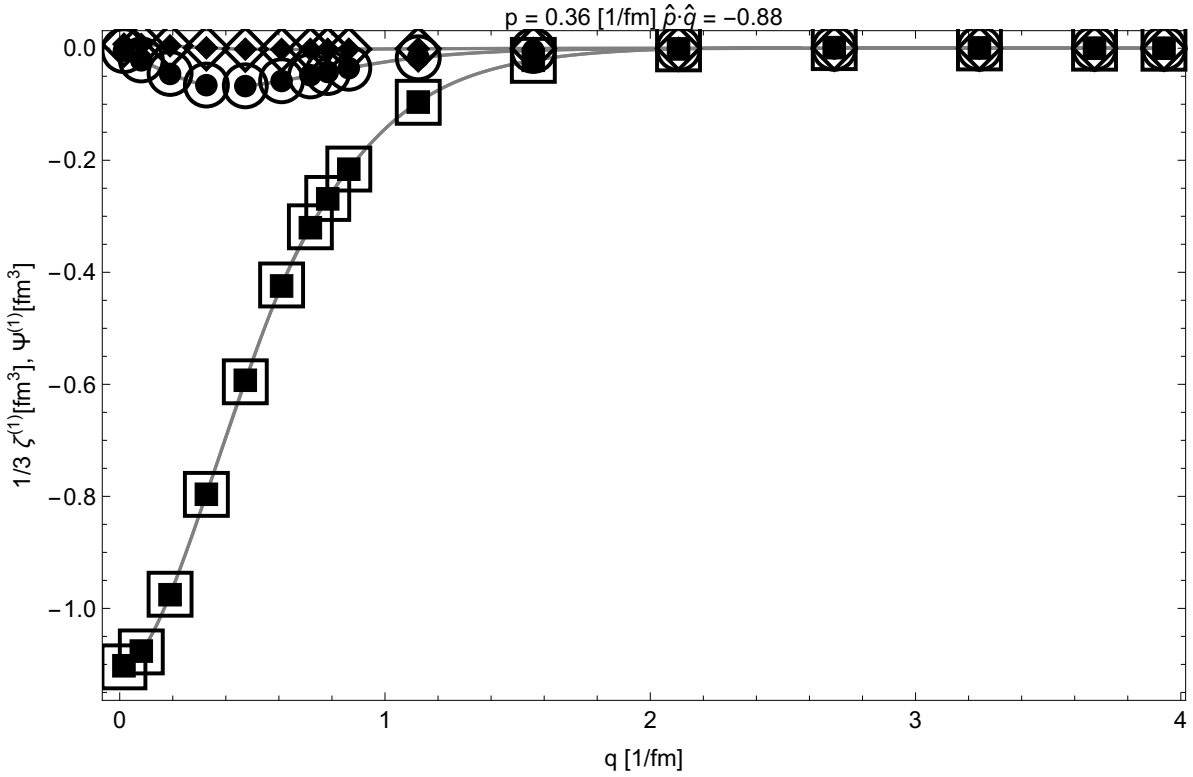

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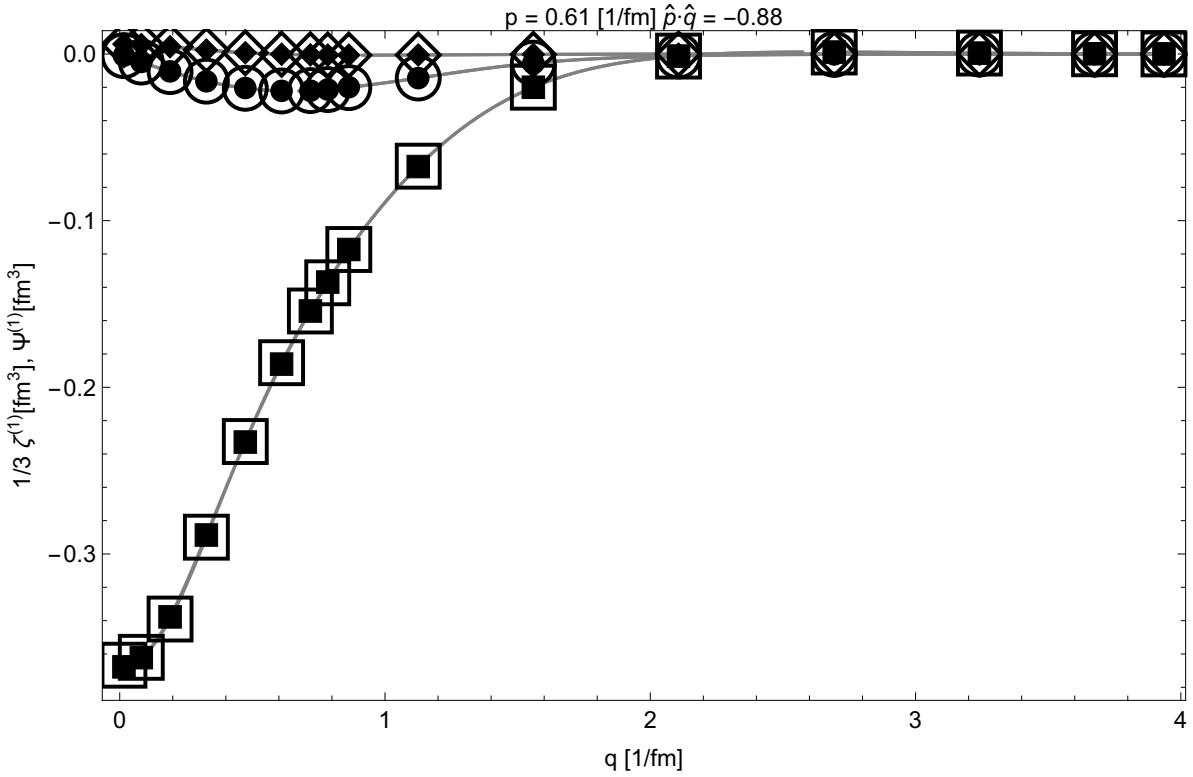

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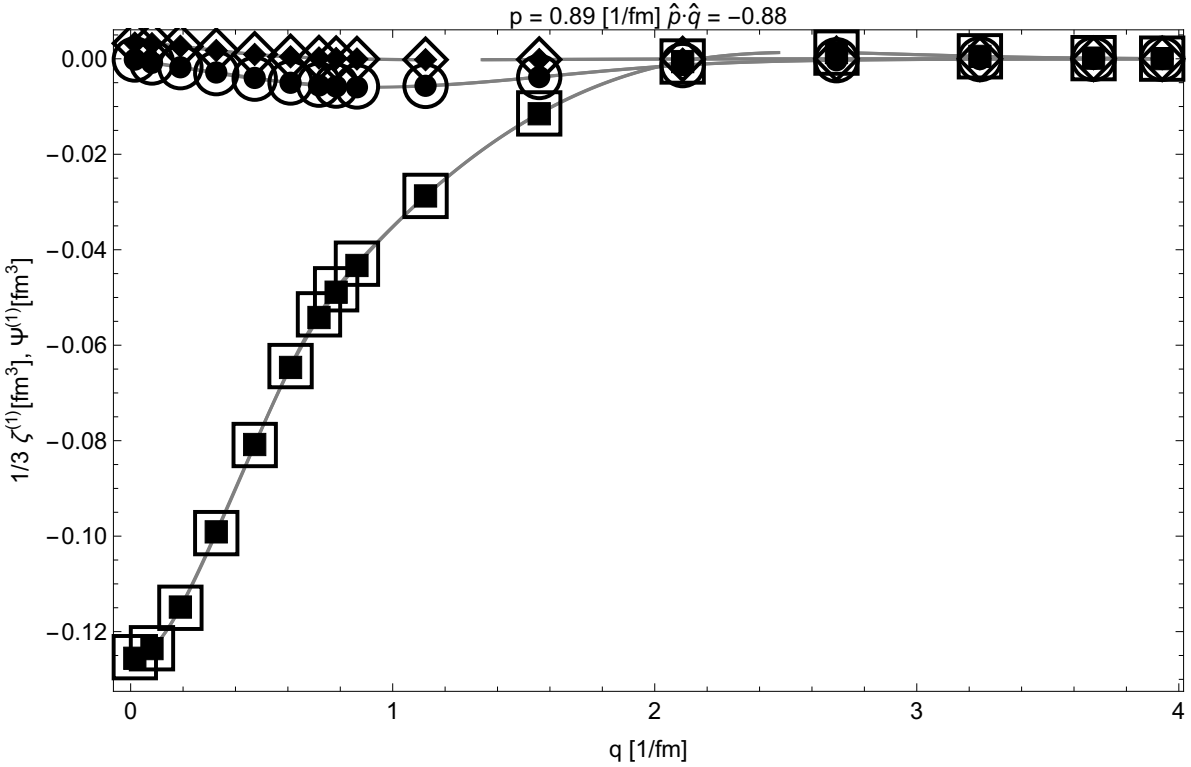

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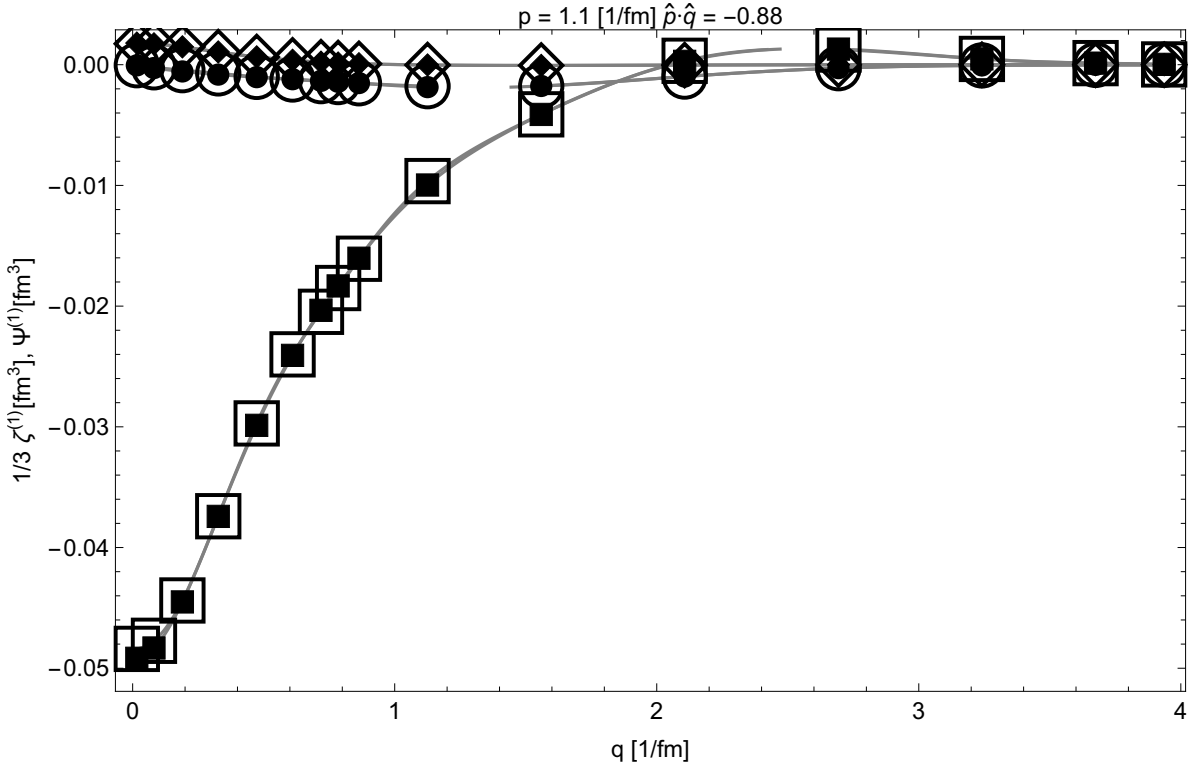

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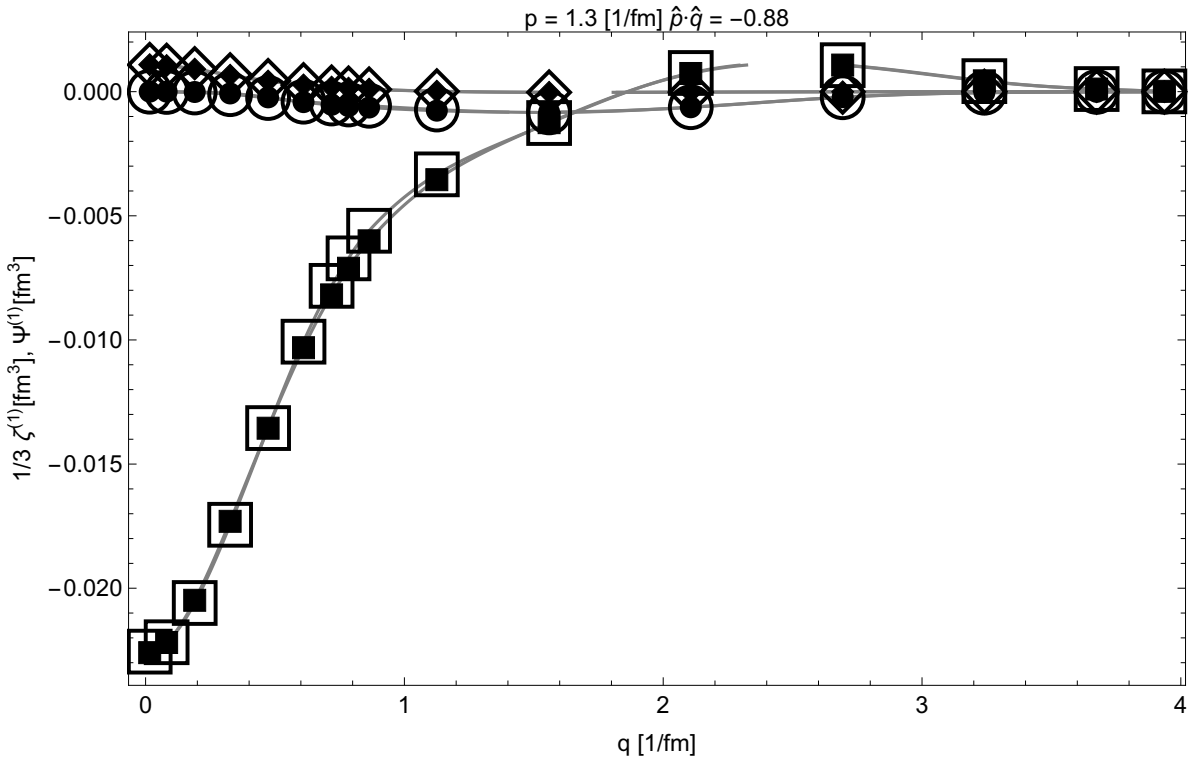

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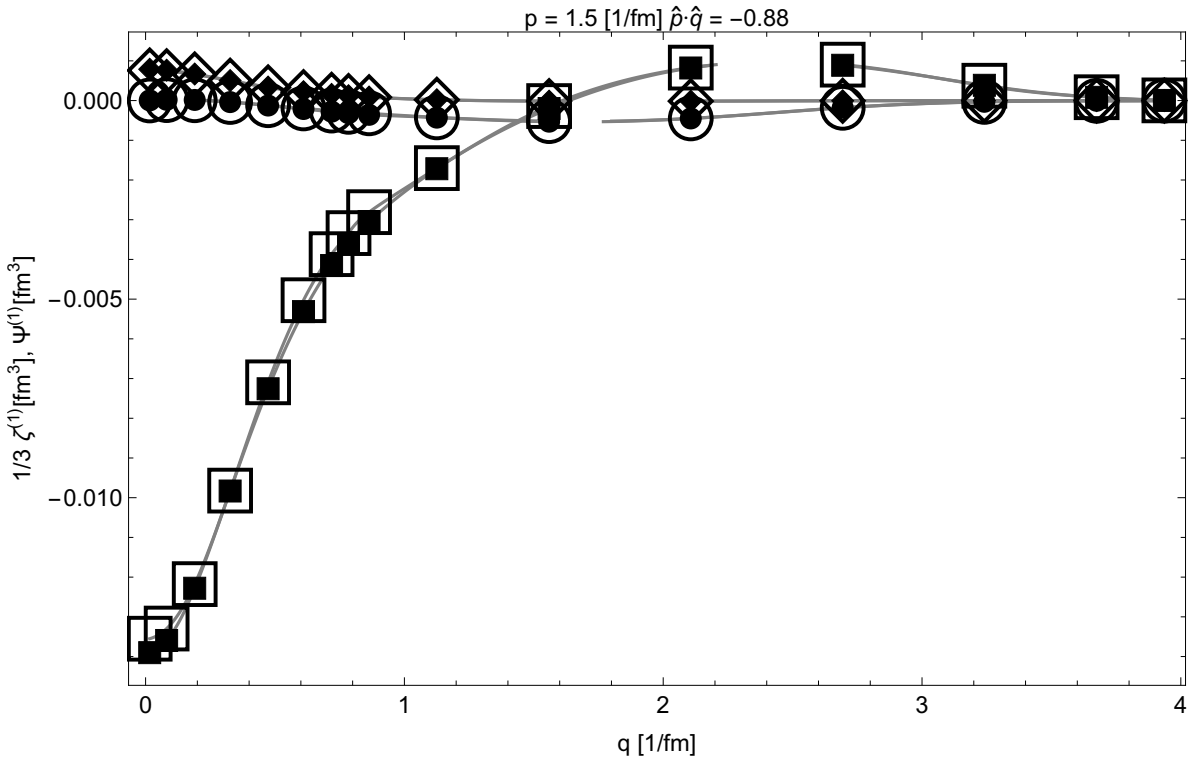

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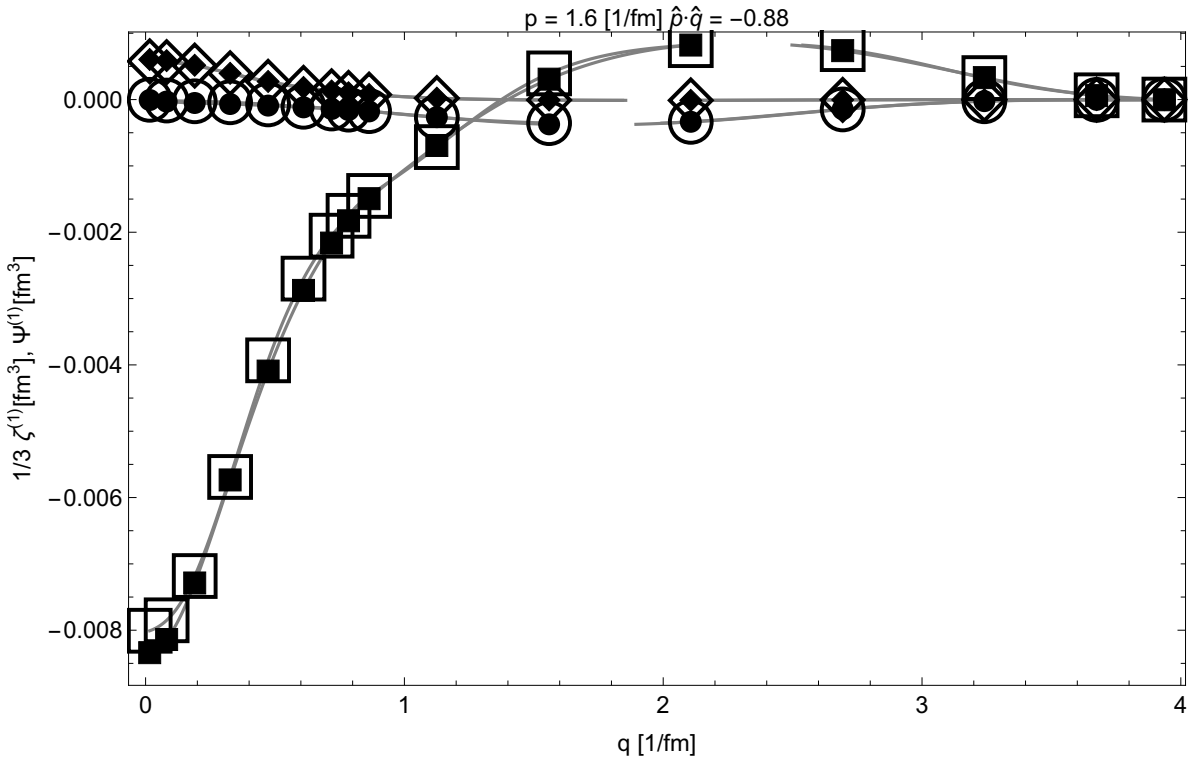

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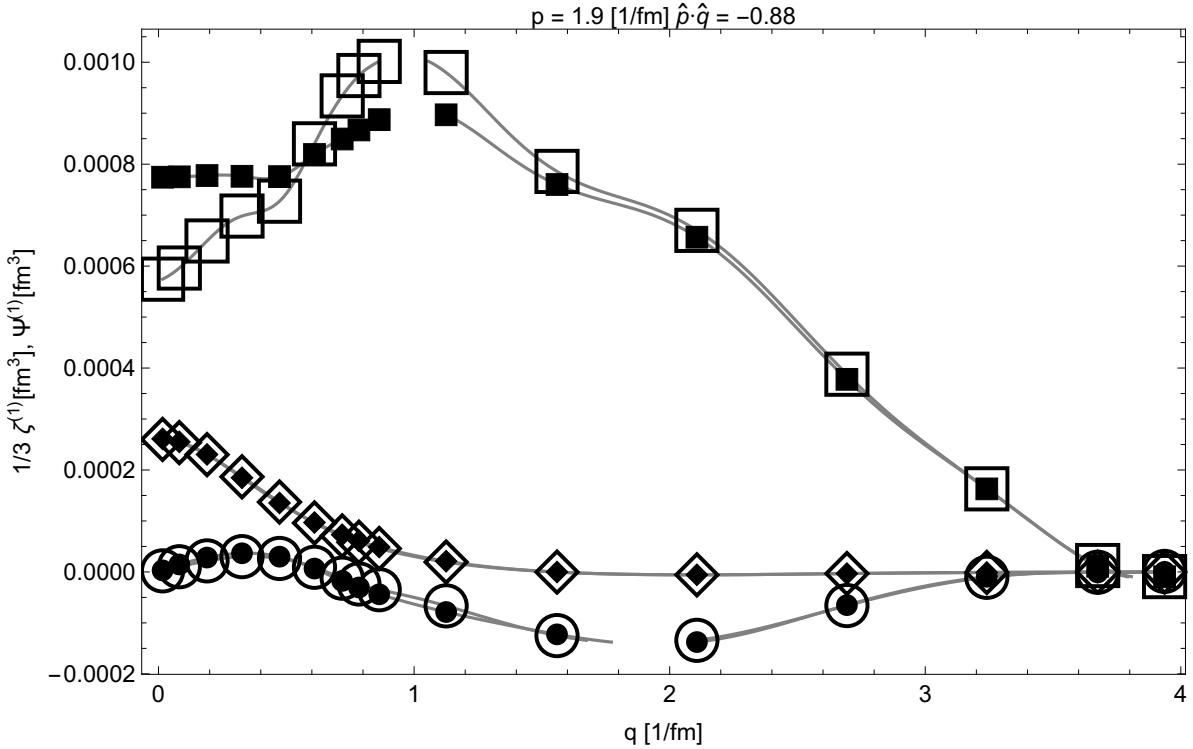

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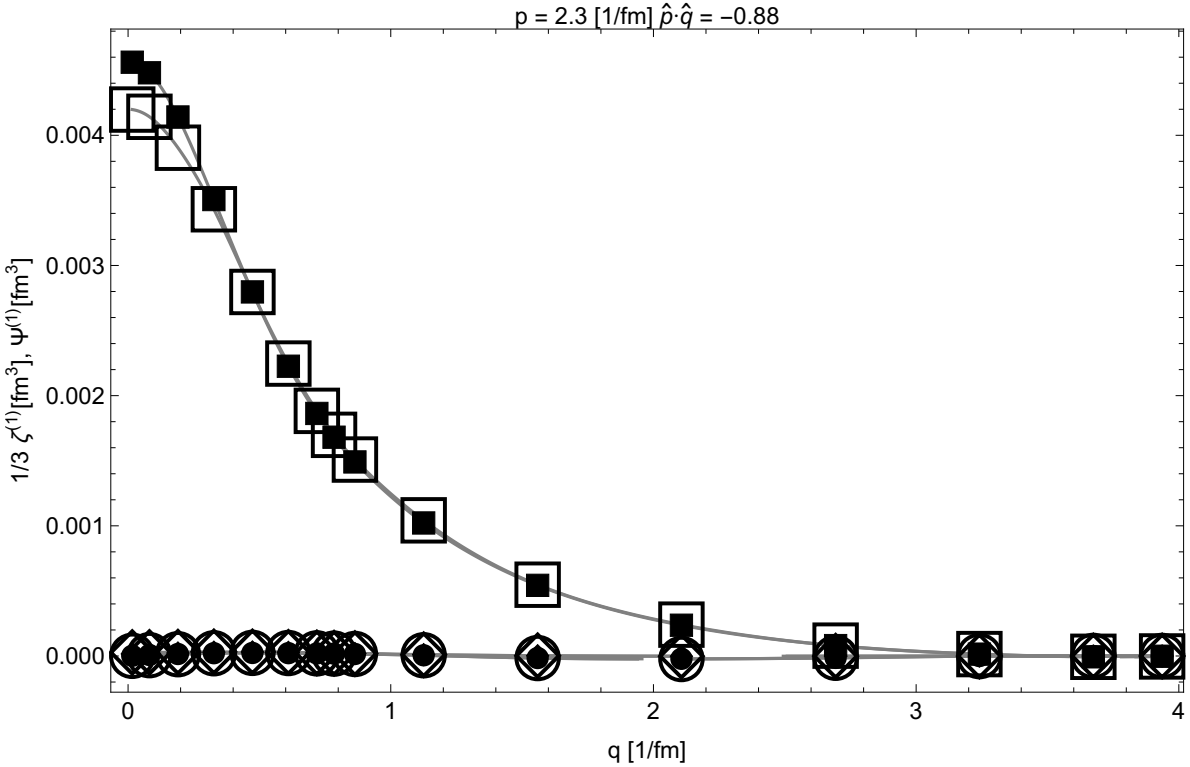

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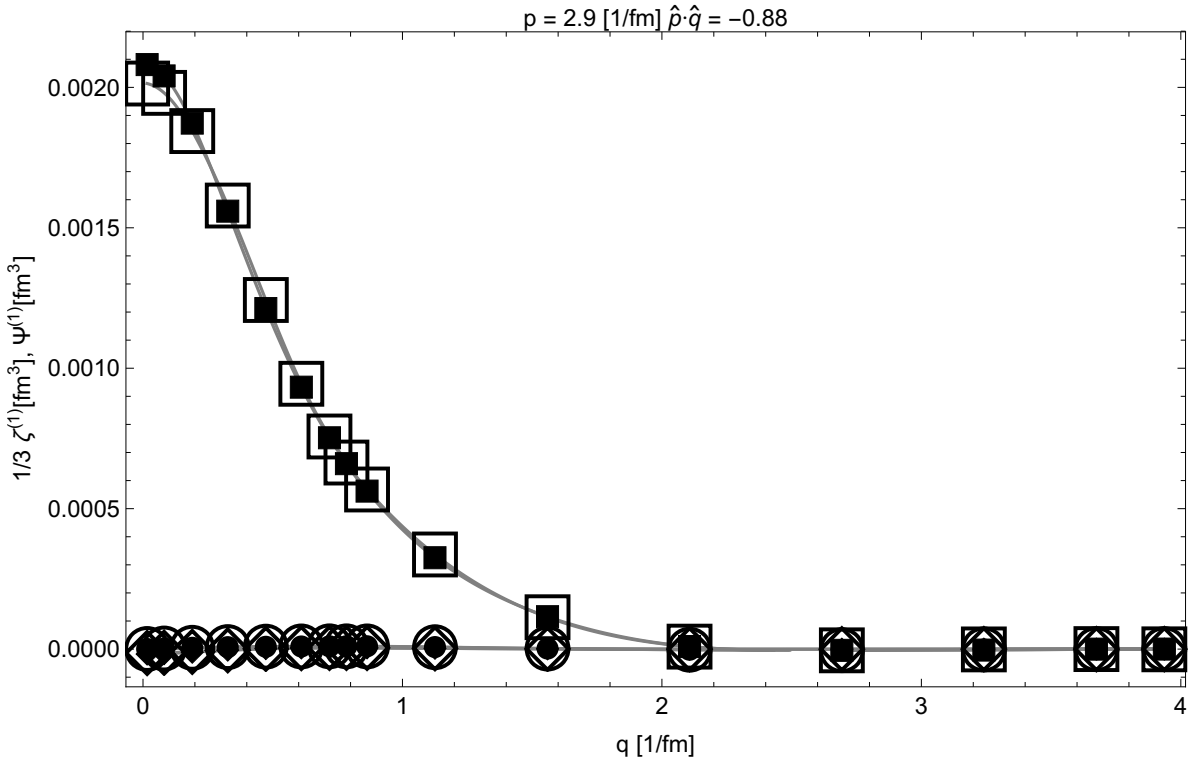

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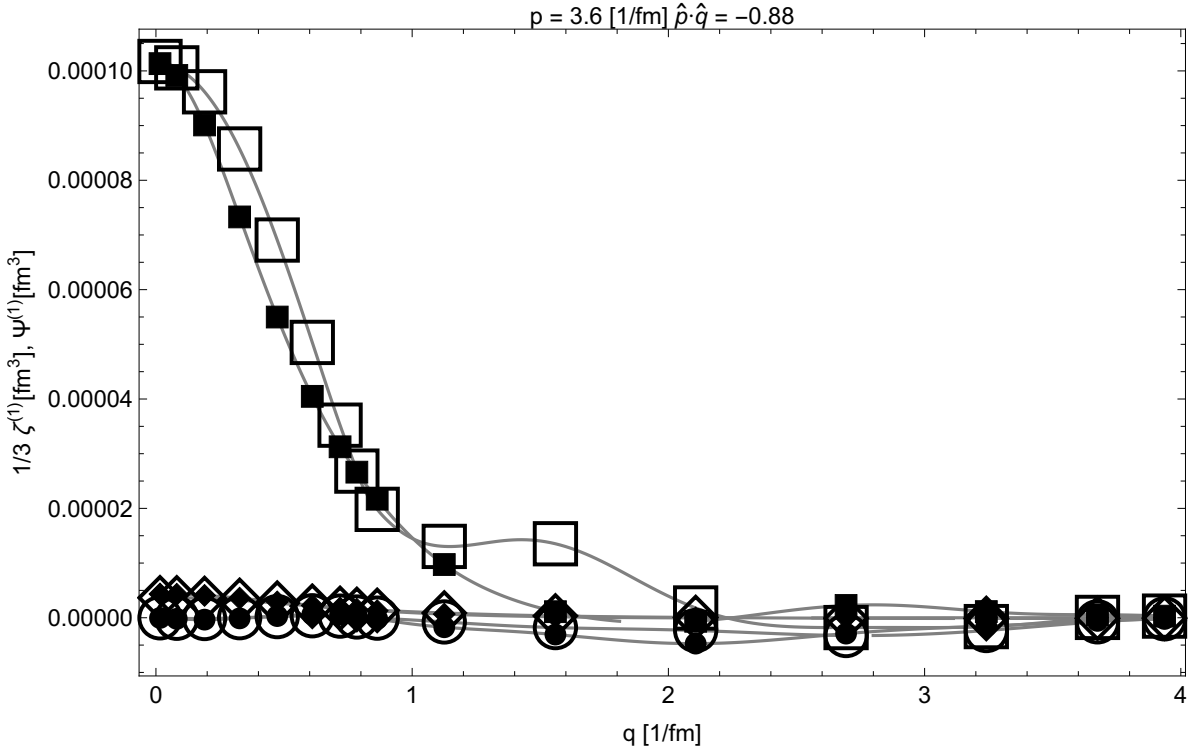

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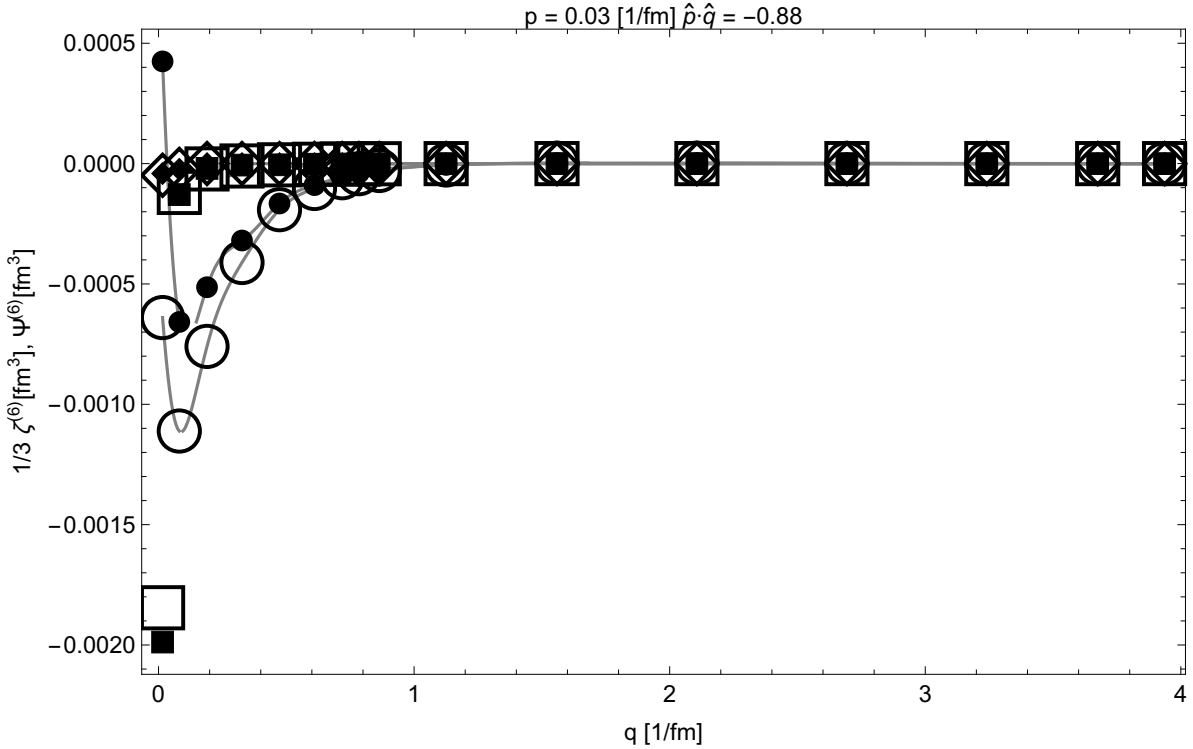

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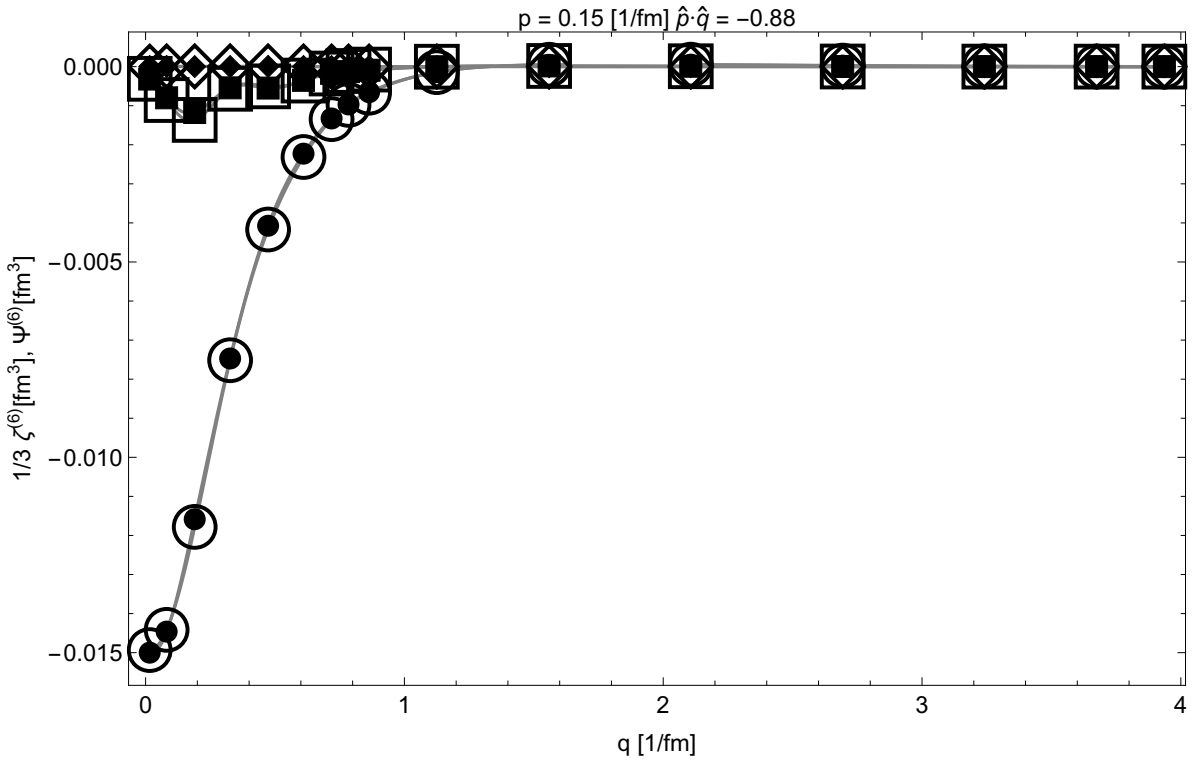

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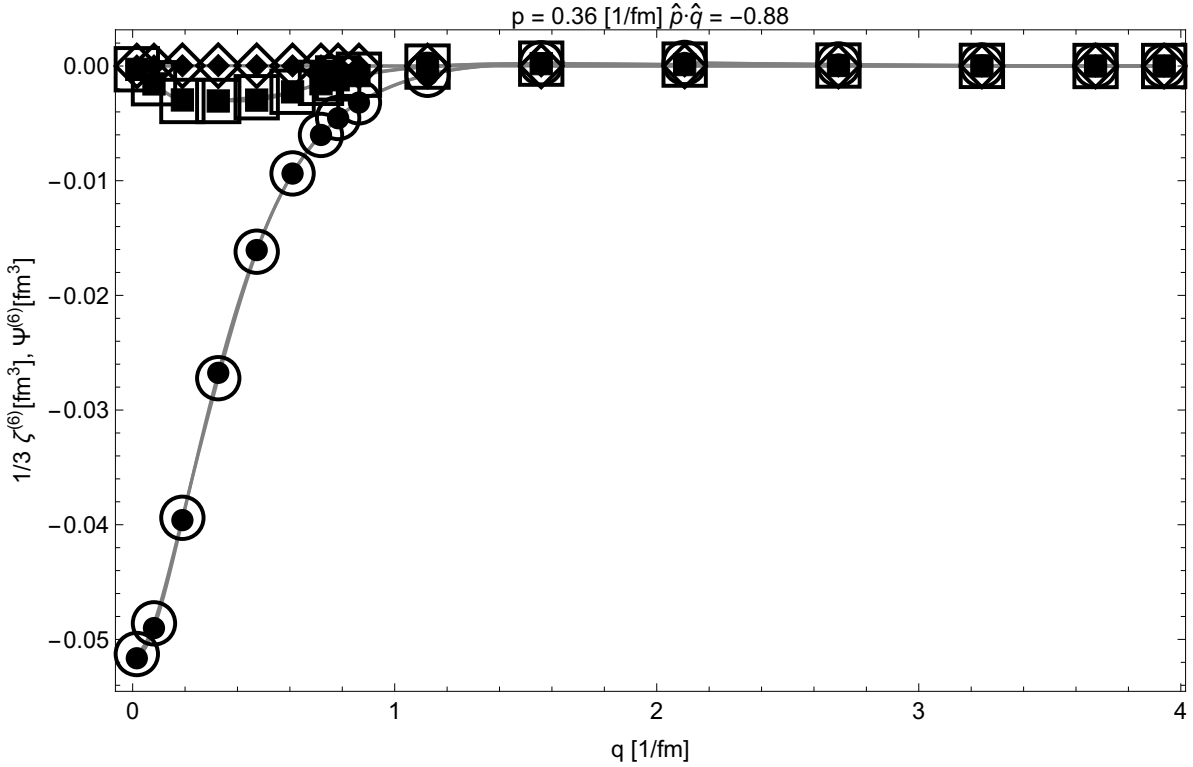

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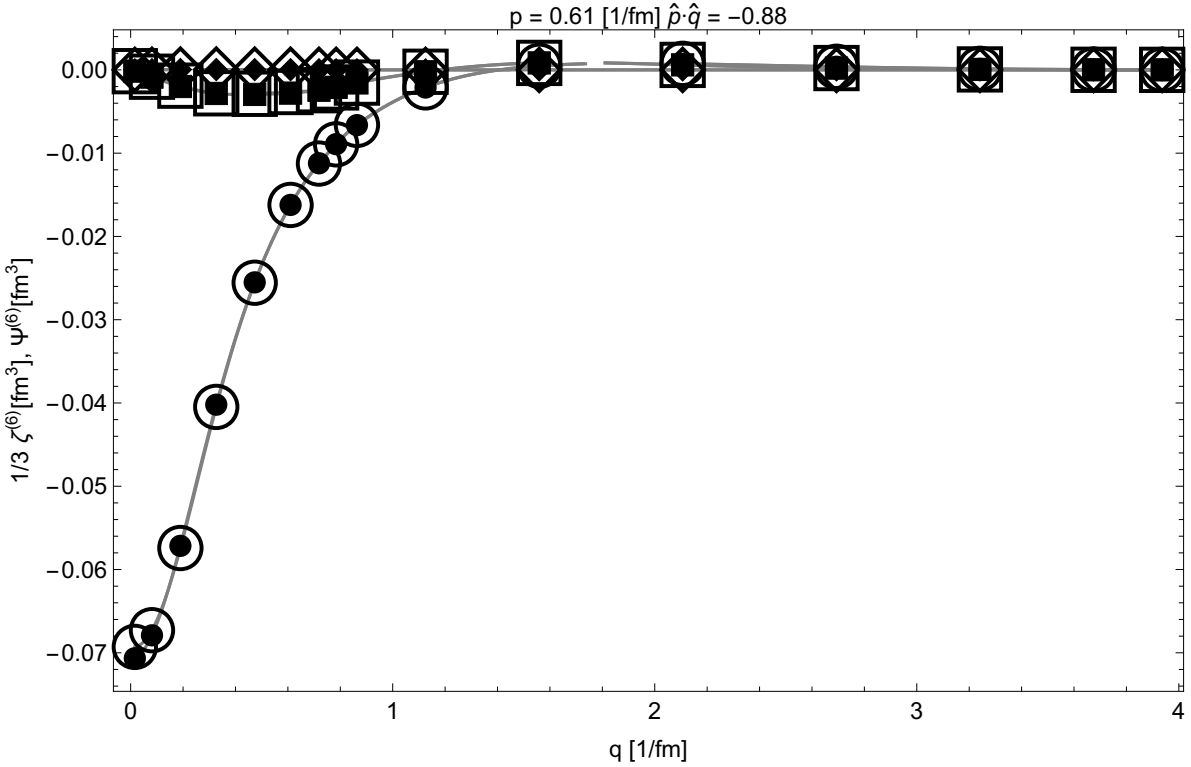

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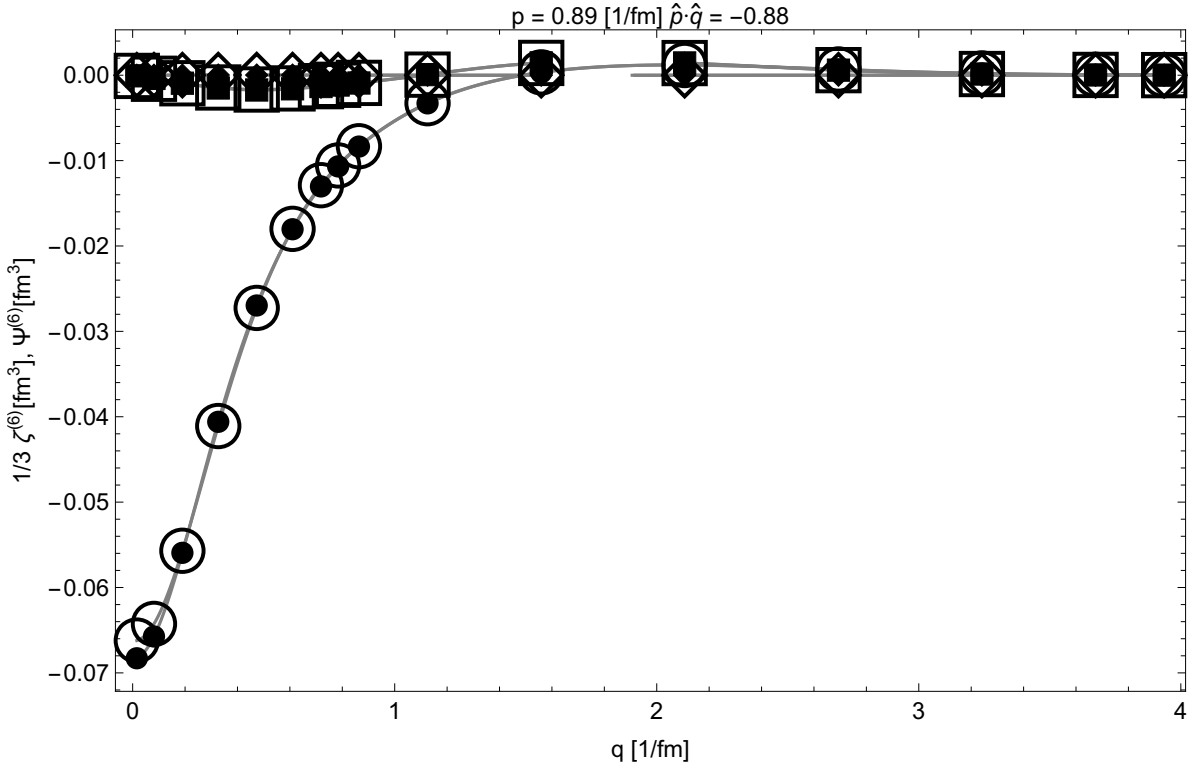

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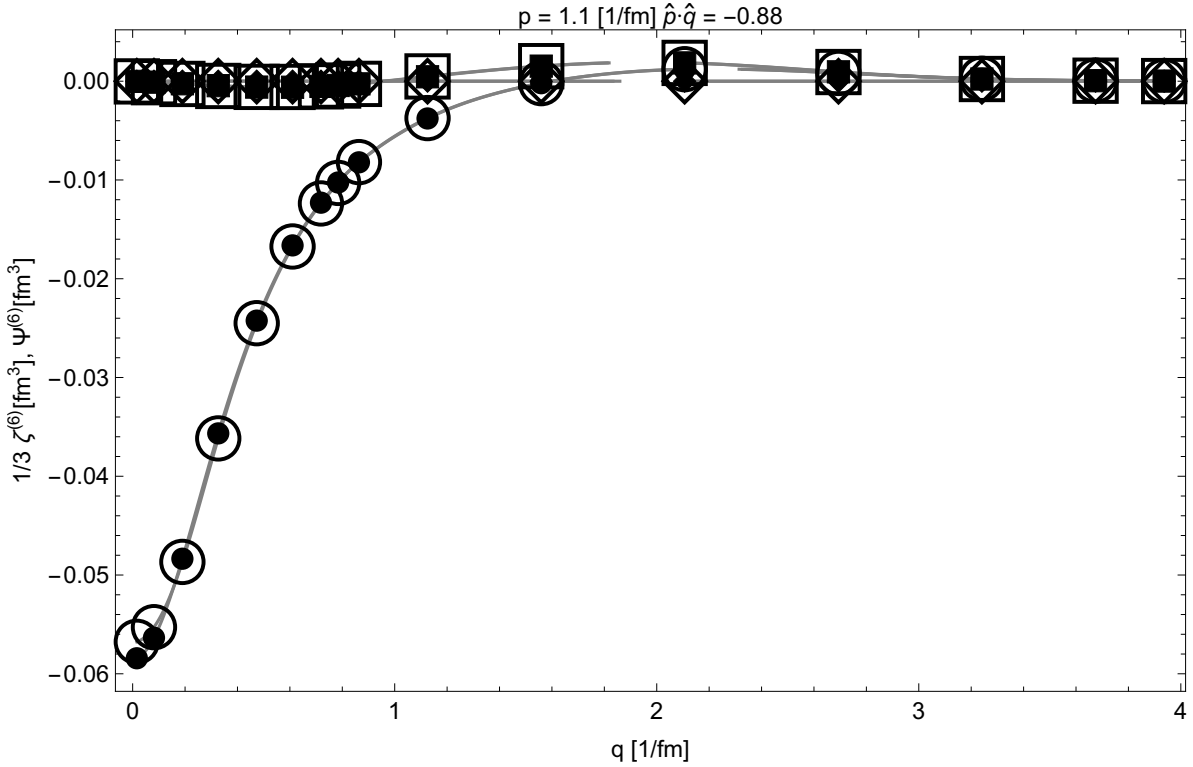

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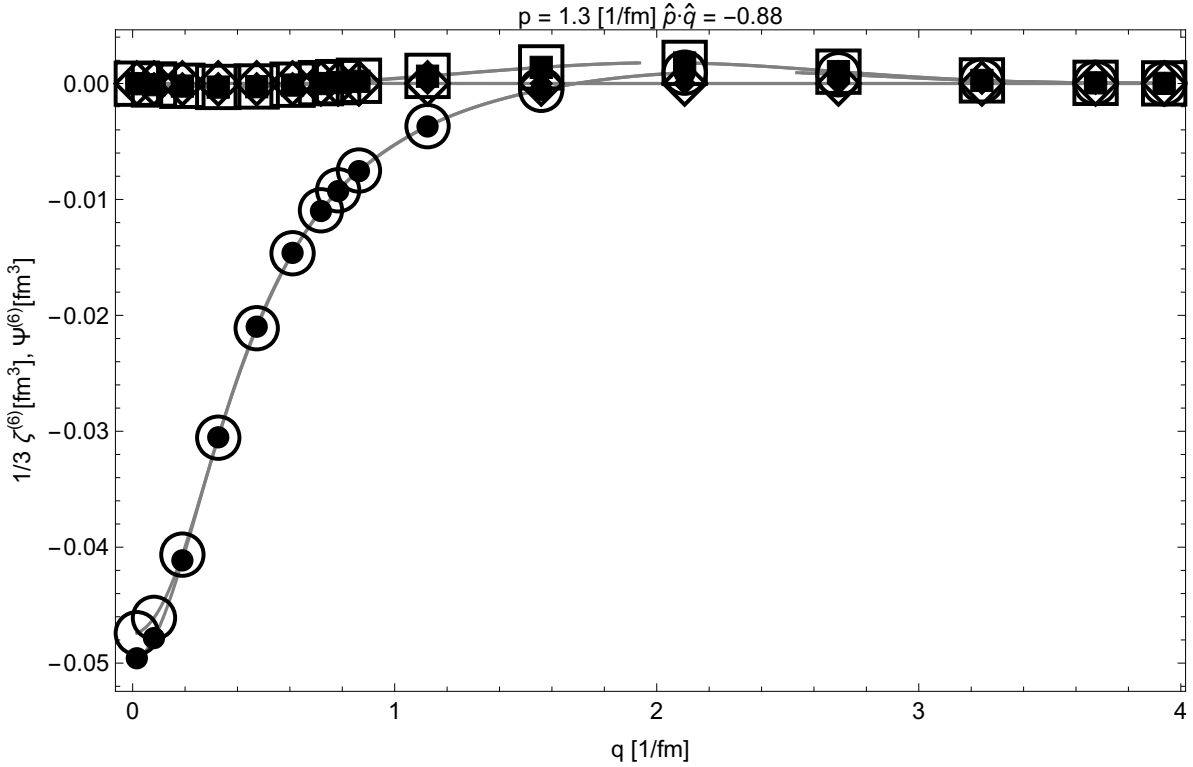

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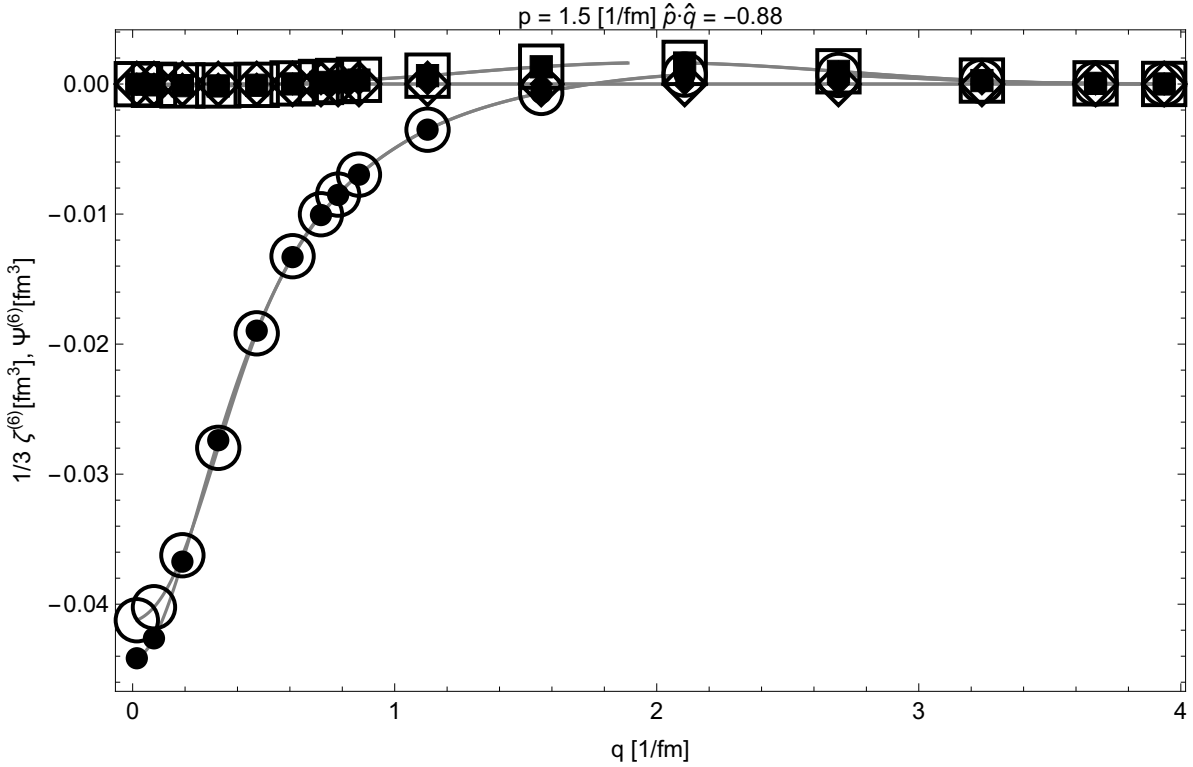

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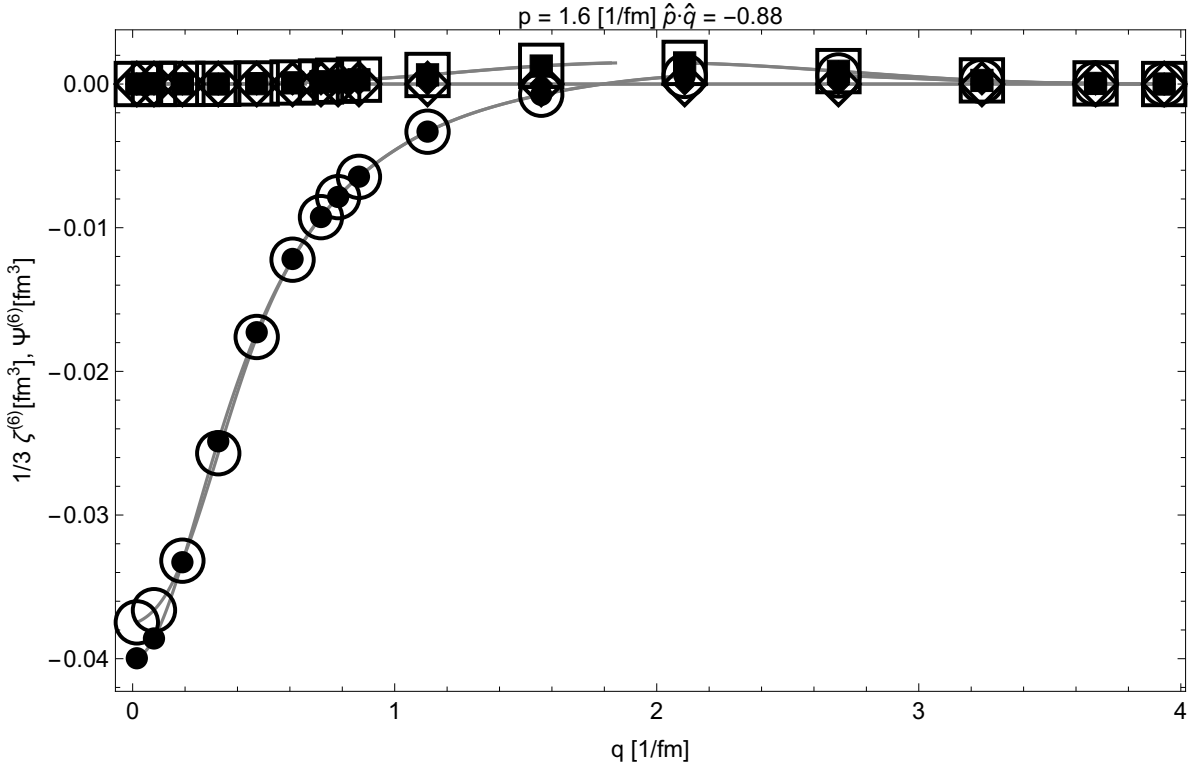

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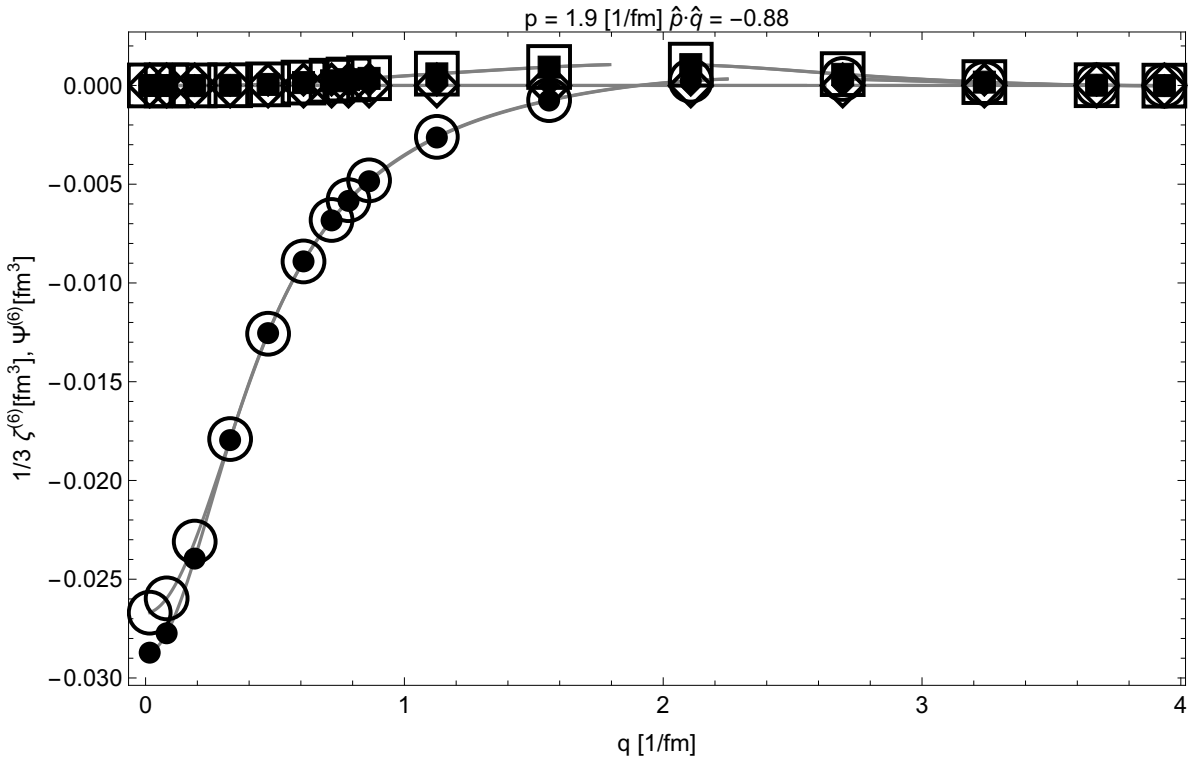

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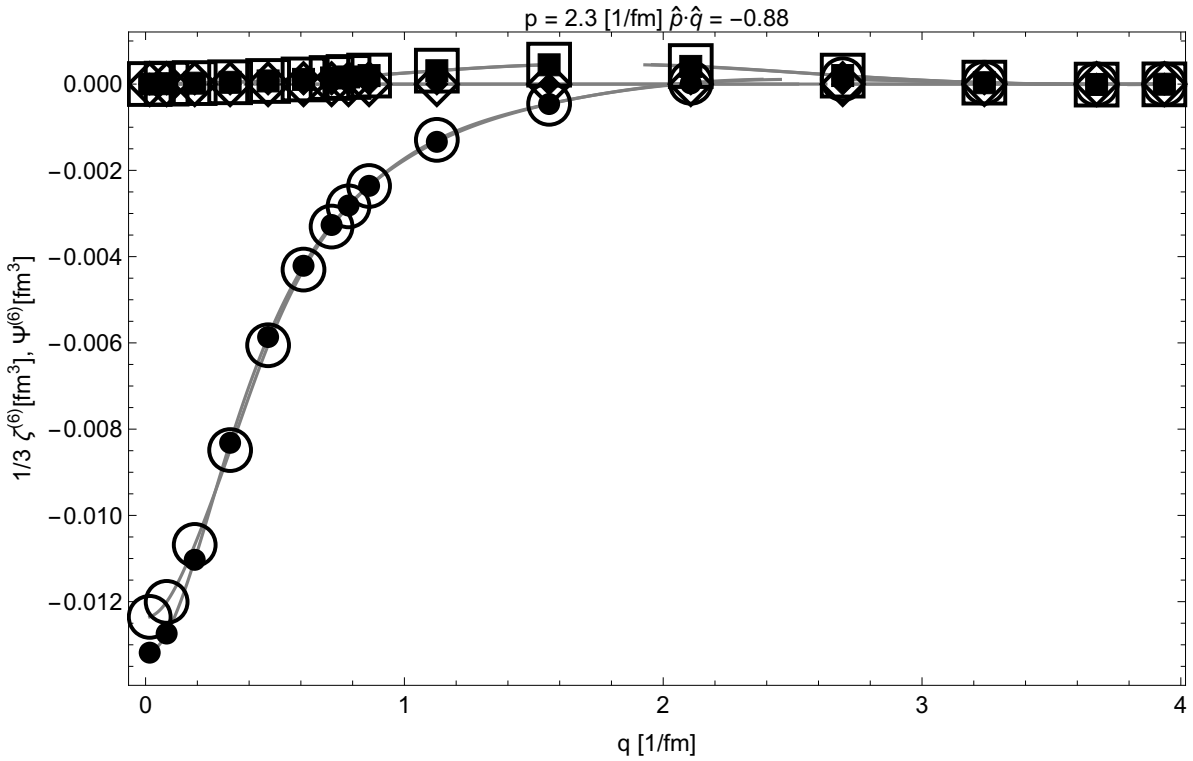

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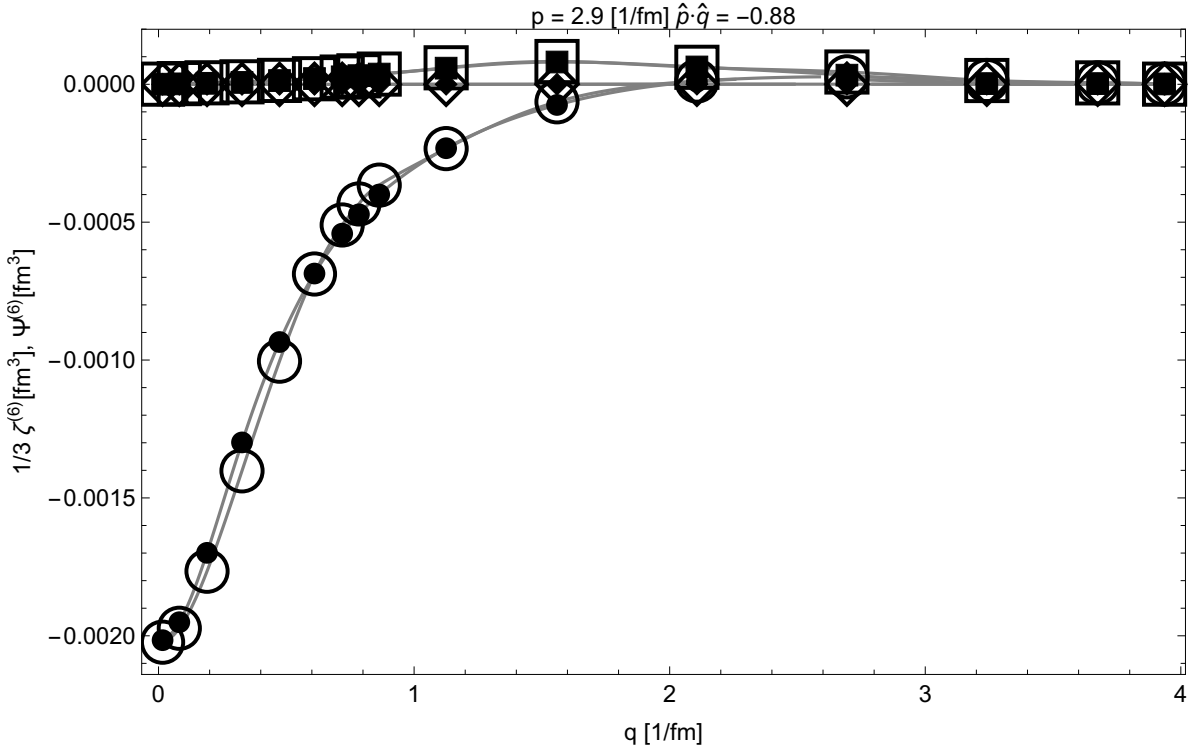

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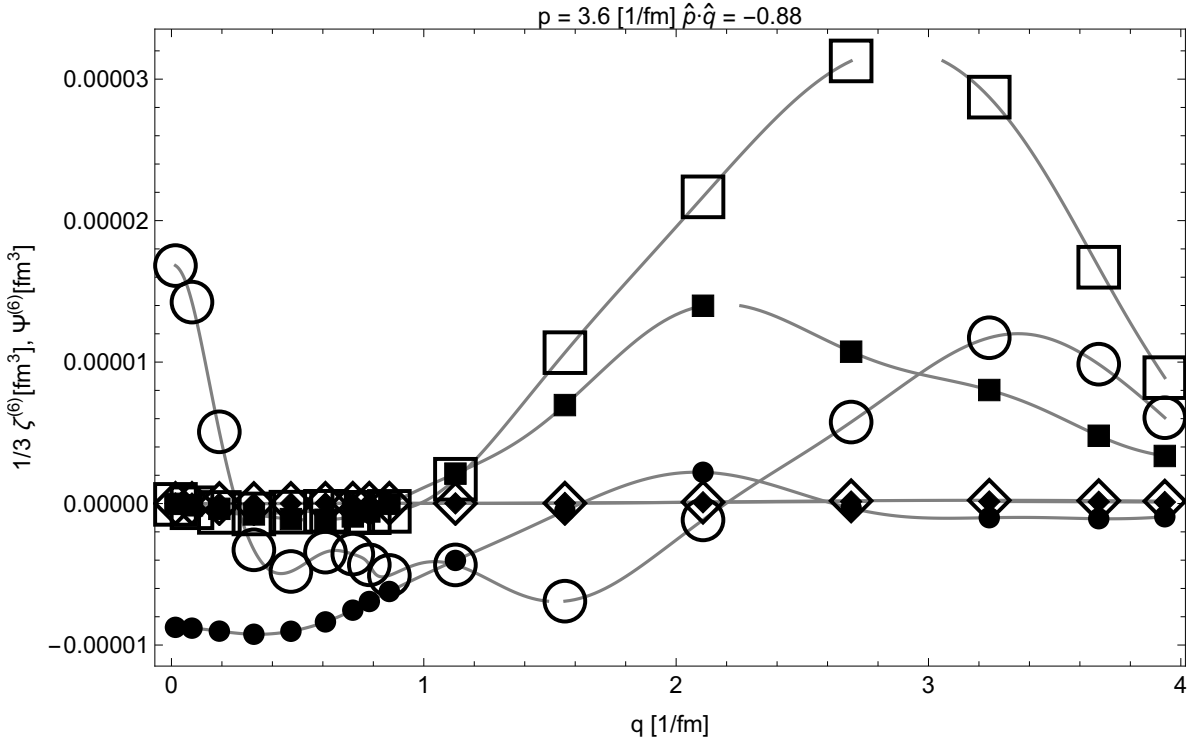

^{3}H BETA DECAY

- Ingrediants: $\boldsymbol{k}'_i$, $\boldsymbol{k}_i$ - momentum of particle i in the final and initial state, $\check{\tau}_i^+$ - isospin raising operator, $\check{\sigma}(i)$ - vector of spin operators.
- Simple models of 1N currents eg.: $\langle \boldsymbol{k}'_i \mid \check{j}_{GTz} \mid \boldsymbol{k}_i \rangle = \check{\tau}_i^+ \check{\sigma}(i)_z$
- $\langle \boldsymbol{k}'_1 \boldsymbol{k}'_2 \boldsymbol{k}'_3 \mid \check{j}(1) \mid \boldsymbol{k}_1 \boldsymbol{k}_2 \boldsymbol{k}_3 \rangle \propto \delta(\boldsymbol{k}'_2 - \boldsymbol{k}_2)\delta(\boldsymbol{k}'_3 - \boldsymbol{k}_3)$ leads to simple expressions
- MELs:

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- More grid points, more precision for newer models of forces
- Easy to calculate matrix elements between ^{3}H and ^{3}He - beta decay
- 3N scattering using the operator form of the 3N scattering amplitude

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The project was financed from the resources of the National Science Center (Poland) under grants 2016/22/M/ST2/00173 and 2016/21/D/ST2/01120. Numerical work was performed on the supercomputers of JSC Jülich.

NARODOWE CENTRUM NAUKI

JÜLICH Forschungszentrum

THANK YOU